Geometry Summer Assignment

This assignment will help you to prepare for Geometry by reviewing some of the things you learned in Algebra 1. If you cannot remember how to complete a specific problem, there is an example at the top of each page. If additional assistance is needed, please use the following websites:

http://www.purplemath.com/modules/index.htm

www.khanacademy.com

This assignment will be due the first day of school.

NAME: ____

Fractions:
Reducing

To reduce a simple fraction, follow the following three steps:

1. **Factor the numerator.**
2. **Factor the denominator.**
3. **Find the fraction mix that equals 1.**

Reduce $\frac{15}{6}$

First: Rewrite the fraction with the numerator and the denominator $\frac{5 x 3}{2 x 3}$ factored:

Second: Find the fraction that equals $\frac{5 x 3}{2 x 3}$ 1. can be written $\frac{5}{2}$ X $\frac{3}{3}$ which in turn can be written $\frac{5}{2}$ X 1 which in turn can be written $\frac{5}{2}$.

Third: We have just illustrated that $\frac{15}{6} = \frac{5}{2}$ Although the left side of the equal sign does not look identical to the right side of the equal sign, both fractions are equivalent because they have the same value. Check it with your calculator. 15 ÷ 6 = 2.5 and 5 ÷ 2 = 2.5. This proves that the fraction $\frac{15}{6}$ can be reduced to the equivalent fraction $\frac{5}{2}$.

Reduce:

1) $\frac{24}{36}$
2) $\frac{14}{18}$
3) $\frac{24}{36}$
4) $\frac{10}{50}$
5) $\frac{-36}{60}$

Operations with Fractions:
Addition and Subtraction

Two fractions can only be added or subtracted if they have the <u>same denominator</u>

For example, it is possible to add $\frac{3}{5}$ and $\frac{1}{5}$ because both fractions have 5 as the denominator.

In this case, we simply add the numerators to find the answer: $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

If fractions do not have the same denominator, you need to find *equivalent* fractions which do

For example, it is not possible to add $\frac{3}{5}$ and $\frac{1}{4}$ without changing each fraction so that they have the same bottom number.

We can use *equivalent fractions* to rewrite each fraction with 20 as the denominator: $\frac{3}{5} = \frac{12}{20}$ and $\frac{1}{4} = \frac{5}{20}$

Now we can see that $\frac{3}{5} + \frac{1}{4} = \frac{12}{20} + \frac{5}{20} = \frac{17}{20}$

Add:

1) $11^1/_3 + 7^1/_6 =$ _____

2) $10^1/_5 + 16^3/_5 =$ _____

3) $1^1/_2 + 16^1/_4 =$ _____

4) $2^4/_5 + 4^3/_5 =$ _____

5) $1^2/_3 + 16^3/_6 =$ _____

6) $6^1/_4 + 10^1/_3 =$ _____

Subtract:

7) $13^1/_6 - 6^1/_3 =$ _____

8) $13^4/_5 - 11^1/_2 =$ _____

9) $11^1/_6 - 4^5/_6 =$ _____

10) $3^3/_4 - 3^1/_2 =$ _____

11) $4^4/_5 - 2^3/_5 =$ _____

12) $10^1/_2 - 6^5/_6 =$ _____

Operations with Fractions: Multiplication

There are 3 simple steps to multiply fractions

1. Multiply the top numbers (the *numerators*).
2. Multiply the bottom numbers (the *denominators*).
3. Simplify the fraction if needed.

$$\frac{1}{2} \times \frac{2}{5}$$

Step 1. Multiply the top numbers:

$$\frac{1}{2} \times \frac{2}{5} = \frac{1 \times 2}{} = \frac{2}{}$$

Step 2. Multiply the bottom numbers:

$$\frac{1}{2} \times \frac{2}{5} = \frac{1 \times 2}{2 \times 5} = \frac{2}{10}$$

Step 3. Simplify the fraction:

$$\frac{2}{10} = \frac{1}{5}$$

1) $^{1}/_{2}$ x $^{2}/_{4}$ = _____

2) $^{1}/_{3}$ x $^{1}/_{4}$ = _____

3) $^{2}/_{5}$ x $^{1}/_{2}$ = _____

4) $^{2}/_{5}$ x $^{1}/_{3}$ = _____

5) $^{4}/_{6}$ x $^{3}/_{5}$ = _____

6) $^{2}/_{3}$ x $^{5}/_{6}$ = _____

7) $^{1}/_{2}$ x $^{1}/_{4}$ = _____

8) $^{5}/_{6}$ x $^{3}/_{6}$ = _____

Operations with Fractions:
Division

There are 3 Steps to Divide Fractions:

Step 1. Turn the second fraction *(the one you want to divide by)* upside-down (this is now a reciprocal).

Step 2. Multiply the first fraction by that reciprocal

Step 3. Simplify the fraction (if needed)

$$\frac{1}{2} \div \frac{1}{6}$$

Step 1. Turn the second fraction upside-down (it becomes a **reciprocal**):

$$\frac{1}{6} \text{ becomes } \frac{6}{1}$$

Step 2. Multiply the first fraction by that **reciprocal**:

$$\frac{1}{2} \times \frac{6}{1} = \frac{1 \times 6}{2 \times 1} = \frac{6}{2}$$

Step 3. Simplify the fraction:

$$\frac{6}{2} = \mathbf{3}$$

1) $^{1}/_{2} \div {}^{4}/_{5}$ = _____

2) $^{3}/_{6} \div {}^{1}/_{3}$ = _____

3) $^{1}/_{3} \div {}^{4}/_{5}$ = _____

4) $^{1}/_{2} \div {}^{1}/_{3}$ = _____

5) $^{1}/_{6} \div {}^{3}/_{5}$ = _____

6) $^{1}/_{4} \div {}^{2}/_{6}$ = _____

7) $^{2}/_{4} \div {}^{1}/_{2}$ = _____

8) $^{4}/_{5} \div {}^{1}/_{6}$ = _____

9) $^{1}/_{3} \div {}^{2}/_{4}$ = _____

10) $^{1}/_{2} \div {}^{3}/_{4}$ = _____

Integers

Adding Integers

1. If the integers have the same sign, add the two numbers and use their common sign
a. 62 + 14 = 76 b. -29 + -13 = -42
2. If the integers have different signs, find the difference between the two values and use the sign of the number that is the greater distance from zero.
a. 15 + -8 = +7 b. 9 + -30 = -21

Subtracting Integers

To subtract an integer, add its opposite
(-8) – (+9) = The opposite of +9 is –9. Change sign to opposite: (-8) + (-9) = -17 using integer addition rules
a. (+7) – (+4) = (+7) + (-4) = +3 b. (-3) – (+8) = (-3) + (-8) = -11
c. (+5) – (-6) = (+5) + (+6) = +11

OR

1. Change double negatives to a positive.
2. Get a sum of terms with like signs and keep the given sign, using the sign in front of the number as the sign of the number.
3. Find the difference when terms have different signs and use the sign of the larger numeral.

a. 7 – (-5) = 7 + 5 = 12 (a. Change double negatives to positive, use integer addition rules)
b. –5 – 9 = -14 (using the signs in front of the numbers, use only addition rules-signs are alike, add and keep the sign)
c. 6 – 7 = -1 (using the signs in front of the numbers, use addition rules-signs are different, subtract and take the sign of the largest numeral)
d. 6 – 7 + 3 – 4 – 2 = 9 – 13 = -4 (Get the sum of the terms with like signs, use addition rules)

1. -5 + -6 =
2. 9 + -4 =
3. -3 + 6 =
4. -4 + -4 =
5. -2 + 8 =
6. -7 - +1 =
7. -9 + 10 =
8. -8 + -5 =
9. 12 + 10 =
10. 13 + -17 =
11. -29 + -11 =
12. -36 - +24 =
13. 42 + -19 =
14. -33 - -42 =
15. 31 - -56 =
16. 65 + 15 =
17. -8 + 10 =
18. 7 + -18 =
19. 75 + -25 =
20. 33 + -22 =
21. 73 - 47 =
22. 86 + -58 =
23. 78 + -30 =
24. 100 + -50 =

Combining Like Terms

What are Like Terms?

The following are like terms because each term consists of a single variable, x, and a numeric coefficient.
`2x, 45x, x, 0x, -26x, -x`

Each of the following are like terms because they are all constants.
`15, -2, 27, 9043, 0.6`

What are Unlike Terms?

These terms are not alike since different variables are used.
`17x, 17z`

These terms are not alike since each y variable in the terms below has a different exponent.
$15y,\ 19y^2,\ 31y^5$

Although both terms below have an x variable, only one term has the y variable, thus these are not like terms either.
`19x, 14xy`

Examples - Simplify Group like terms together first, and then simplify.

$\mathbf{2x^2 + 3x - 4 - x^2 + x + 9}$

$2x^2 + 3x - 4 - x^2 + x + 9$
$= (2x^2 - x^2) + (3x + x) + (-4 + 9)$
$= \mathbf{x^2 + 4x + 5}$

$\mathbf{10x^3 - 14x^2 + 3x - 4x^3 + 4x - 6}$

$10x^3 - 14x^2 + 3x - 4x^3 + 4x - 6$
$= (10x^3 - 4x^3) + (-14x^2) + (3x + 4x) - 6$
$= \mathbf{6x^3 - 14x^2 + 7x - 6}$

Directions: *Simplify each expression below* by combining like terms.

1) $-6k + 7k$

2) $12r - 8 - 12$

3) $n - 10 + 9n - 3$

4) $-4x - 10x$

5) $-r - 10r$

6) $-2x + 11 + 6x$

7) $-v + 12v$

8) $x + 2 + 2x$

9) $5 + x + 2$

10) $2x^2 + 13 + x^2 + 6$

11) $2x + 3 + x + 6$

12) $2x^3 + 3x + x^2 + 4x^3$

Distributive Property

In algebra, the use of parentheses is used to indicate operations to be performed. For example, the expression $4(2x-y)$ indicates that *4 times the binomial* $2x-y$ is $8x - 4y$

Additional Examples:

1. $2(x+y) = 2x+2y$
2. $-3(2a+b-c) = -3(2a)-3(b)-3(-c)=-6a-3b+3c$
3. $3(2x+3y) = 3(2x)+3(2y)=6x+9y$

1. $3(4x + 6) + 7x =$

2. $7(2 + 3x) + 8 =$

3. $9 + 5(4x + 4) =$

4. $12 + 3(x + 8) =$

5. $3(7x + 2) + 8x =$

6. $6m + 3(2m + 5) + 7 =$

7. $5(m + 9) - 4 + 8m =$

8. $3m + 2(5 + m) + 5m =$

9. $6m + 14 + 3(3m + 7) =$

10. $4(2m + 6) + 3(3 + 5m) =$

Evaluating Expressions

Simplify the expression first. Then evaluate the resulting expression for the given value of the variable.

Example $3x + 5(2x + 6) =$ _____ if $x = 4$
$3x + 10x + 30 =$
$13x + 30 =$
$13(4) + 30 =$ <u>82</u>

1. $y + 9 - x =$ ____; if $x = 1$, and $y = 3$

5. $7(7 + 5m) + 4(m + 6) =$ _____ if $m = 1$

2. $8 + 5(9 + 4x) =$ _____ if $x = 2$

6. $2(4m + 5) + 8(3m + 1) =$ _____ if $m = 3$

3. $6(4x + 7) + x =$ _____ if $x = 2$

7. $5(8 + m) + 2(7m - 7) =$ ______ if $m = 3$

4. $9(2m + 1) + 2(5m + 3) =$ _____ if $m = 2$

8. $y \div 2 + x =$ ____; if $x = 1$, and $y = 2$

Solving Equations

An equation is a mathematical statement that has two expressions separated by an equal sign. The expression on the left side of the equal sign has the same value as the expression on the right side. To *solve an equation* means to determine a numerical value for a variable that makes this statement true by isolating or moving everything except the variable to one side of the equation. To do this, combine like terms on each side, then add or subtract the same value from both sides. Next, clear out any fractions by multiplying **every** term by the denominator, and then divide every term by the same nonzero value. Remember to keep both sides of an equation equal, you must do exactly the same thing to each side of the equation.

Examples:

a. $x + 3 = 8$

$\underline{\quad -3 \quad -3}$

$x \quad = 5$

3 is being added to the variable, so to get rid of the added 3, we do the opposite, subtract 3.

b. $5x - 2 = 13$

$\underline{\quad +2 \quad +2}$

$5x \quad = 15$

$\frac{5x}{5} = \frac{15}{5}$

$x = 3$

First, undo the subtraction by adding 2.

Then, undo the multiplication by dividing by 5.

Solve

1.) $-7 - 4x = -31$

2.) $-7x + 7 = -70$

3.) $\frac{5x}{2} + 18 = 28$

4.) $2x + 7 = 31$

5.) $-4x + 2 = -10$

6.) $8 - 7x = -13$

7.) $-3x - 1 = 17$

8.) $5x + 5 = 35$

Solving Equations

If an equation has two terms with a variable, get the variables combined into one term by moving the variable with the smaller coefficient. To do this, add or subtract the same variable from both sides Remember, to keep both sides of an equation equal, we must do exactly the same thing to each side of the equation.

$$\begin{array}{rcl} 4x + 5 &=& x - 4 \\ \underline{-x \qquad\quad} && \underline{-x \quad\;\;} \\ 3x + 5 &=& -4 \end{array}$$

Then proceed as before.

$$\begin{array}{rcl} 3x + 5 &=& -4 \\ \underline{\quad -5} && \underline{-5} \\ \frac{3x}{3} &=& \frac{-9}{3} \\ x &=& -3 \end{array}$$

Solve

1.) 5x + 8 = - 2 + 6x

2.) - 6 + 5x = 2x + 15

3.) 10 + 4x = 7x - 17

4.) x + 2 = 6x - 13

5.) 7x - 7 = - 19 + 6x

6.) - 2x + 4 = - 7x - 6

7.) 10 - 3x = - 2x + 3

8.) 9 + x = - 3x - 3

Ratio

A ratio is a statement of how two numbers compare. We use ratios to make comparisons between two things. When we express ratios in words, we use the word "to" -- we say "the ratio of something to something else". Multiplying or dividing each term by the same nonzero number will give an equal ratio. For example, the ratio 2:4 is equal to the ratio 1:2. To tell if two ratios are equal, use a calculator and divide. If the division gives the same answer for both ratios, then they are equal.

Write each ratio in simplest form.

1. 63 to 14	2. 22:40	3. $\frac{17}{17}$	4. 8:13	5. 50 to 55
6. 60 to 60	7. 21:30	8. 20 to 60	9. 20:16	10. $\frac{26}{28}$

Write five equivalent ratios for each ratio.

1. 45:24	2. 4 to 34	3. 18 to 5
4. 8:4	5. 16 to 12	6. 11:20

Proportion

A proportion is a name we give to a statement that two ratios are equal. It can be written in two ways:

- two equal fractions, $\frac{a}{b} = \frac{c}{d}$
- using a colon, **a:b = c:d**

When two ratios are equal, then the cross products of the ratios are equal.

That is, for the proportion, **a:b = c:d , a** x **d = b** x **c**

Determine the missing value:

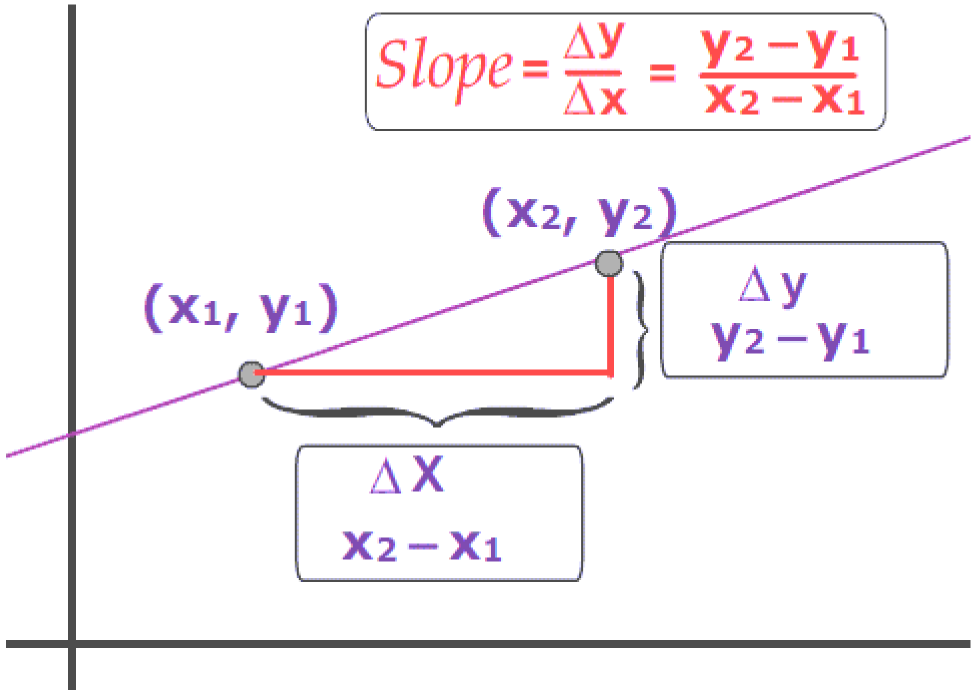
1. $\frac{15}{p} = \frac{20}{8}$	2. $\frac{s}{10} = \frac{84}{20}$	3. $\frac{3}{y} = \frac{9}{12}$
4. $\frac{4}{12} = \frac{v}{3}$	5. $\frac{12}{28} = \frac{p}{21}$	6. $\frac{20}{12} = \frac{f}{9}$
7. $\frac{5}{9} = \frac{z}{27}$	8. $\frac{1}{4} = \frac{4}{q}$	9. $\frac{4}{h} = \frac{1}{2}$

State whether the ratios are proportional:

1. $\frac{35}{20} = \frac{7}{4}$	2. $\frac{3}{8} = \frac{32}{12}$	3. $\frac{5}{13} = \frac{40}{48}$	4. $\frac{9}{24} = \frac{3}{8}$
5. $\frac{52}{28} = \frac{40}{16}$	6. $\frac{10}{9} = \frac{20}{18}$	7. $\frac{10}{45} = \frac{2}{9}$	8. $\frac{8}{9} = \frac{2}{36}$

Slope of a Line

The slope of a line characterizes the general direction in which a line points. To find the slope, you divide the difference of the y-coordinates of a point on a line by the difference of the x-coordinates.



Example: Find the slope of a line through the points (4,3) and (1,2).

Starting with the point (4,3) Slope = $\frac{y^2-y^1}{x^2-x^1} = \frac{3-2}{4-1} = \frac{1}{3}$

OR you can start with the point (1,2)

Slope = $\frac{y^2-y^1}{x^2-x^1} = \frac{2-3}{1-4} = \frac{-1}{-3} = \frac{1}{3}$

1) What is the slope of a line that goes through the points (-10,3) and (7,9) ?

2) A line passes through (2,10) and (8,7). What is its slope?

3) A line passes through (12,11) and (9,5). What is its slope?

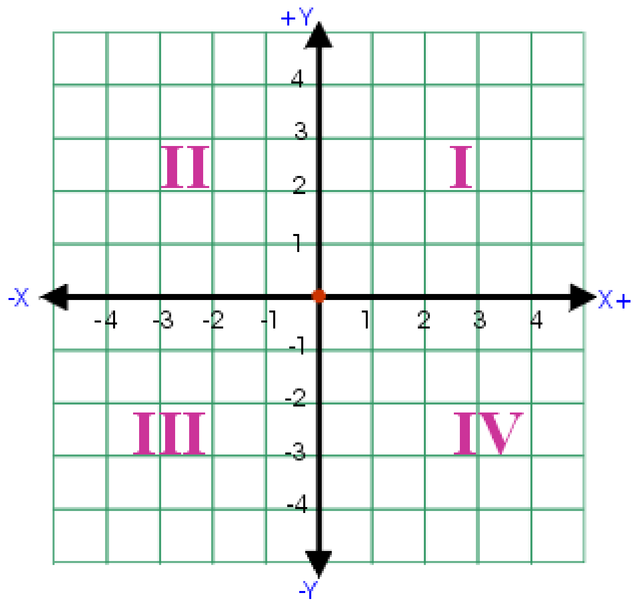
4) What is the slope of a line that goes through (4,2) and (4,5)?

The Coordinate Plane

This is a **coordinate plane**. It has two axes and four quadrants. The two number lines form the axes. The horizontal number line is called the **x-axis** and the vertical number line is called the **y-axis**.

The center of the coordinate plane is called the **origin**. It has the coordinates of (0,0).

Locations of points on the plane can be plotted when one coordinate from each of the axes are used. This set of x and y values are called **ordered pairs**.



State the quadrant or axis that each point lies in.

1) $J\,(5, 10)$ ______________

2) $G(-6, 8)$ ______________

3) $D(-8, -4)$ ______________

4) $A\,(-8, 1)$ ______________

5) $I\,(1, 9)$ ______________

6) $F\,(9, 0)$ ______________

7) $C\,(0, 5)$ ______________

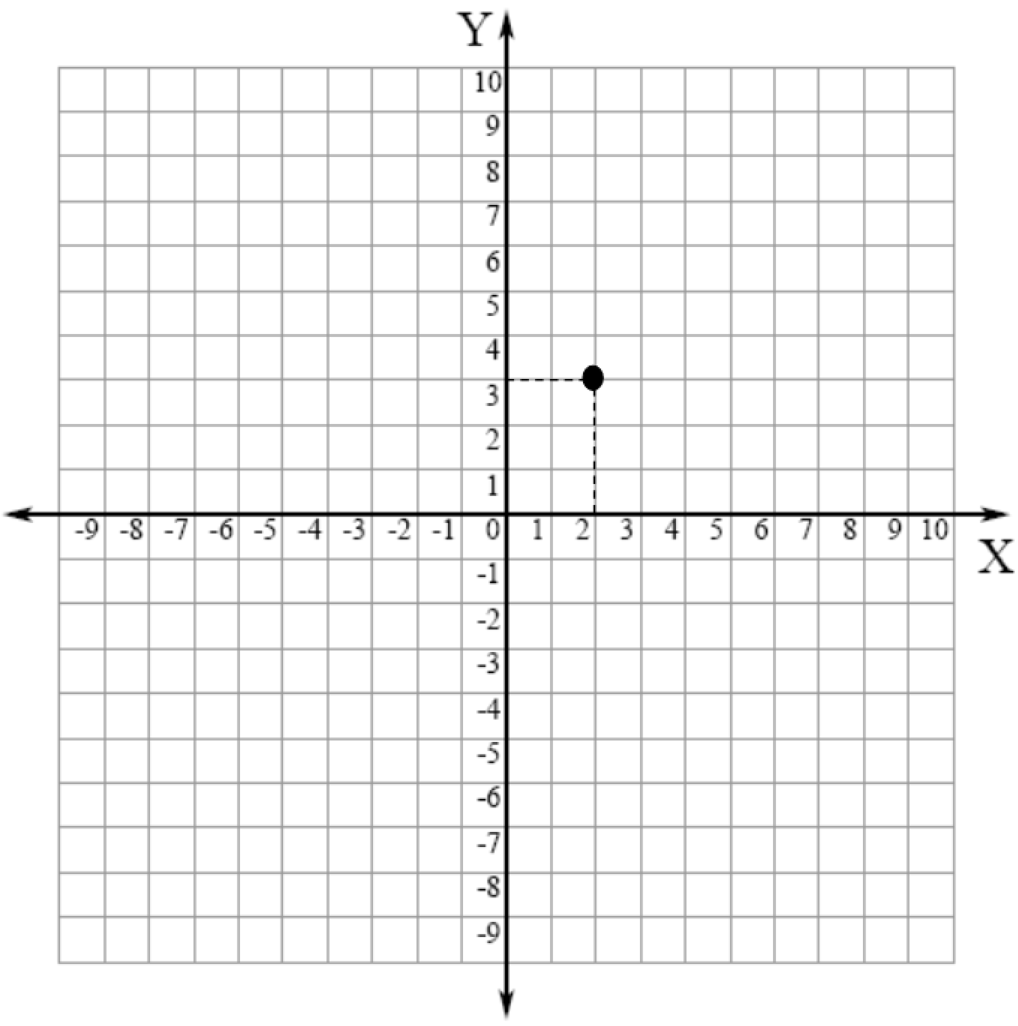
8) $H\,(6, -9)$ ______________

9) $E\,(6, 0)$ ______________

10) $B\,(1, 1)$ ______________

Plotting Points

The first coordinate of a plotted point is called the '**x**' coordinate. The 'x' coordinate is the horizontal distance from the origin to the plotted point. The second coordinate of a plotted point is called the '**y**' coordinate. The 'y' coordinate is the vertical distance from the origin to the plotted point. So, to locate the point: (2, 3) on our graph below, we start at the origin and move 2 units horizontally and 3 units vertically. When locating points, **positive** 'x' values are to the **right** of the origin, while **negative** 'x' values are to the **left** of the origin. Also, positive 'y' values are above the origin, while negative 'y' values are below the origin.



Plot each of the points on the graph:

(1) Point D at (0, 10)
(2) Point J at (-1, 6)
(3) Point O at (-8, 1)
(4) Point B at (-9, -3)
(5) Point E at (-4, -8)
(6) Point F at (5, 6)
(7) Point S at (-8, 2)
(8) Point H at (6, 8)
(9) Point P at (-9, -10)
(10) Point G at (-7, 9)
(11) Point Z at (-7, -5)
(12) Point Y at (0, -8)

Simplifying Radicals

Step 1) Find the largest perfect square that is a factor of the radicand
Step 2) Rewrite the radical as a product of the perfect square and its matching factor
Step 3) Simplify

Example: $\sqrt{8}$

Step 1) 4 is the largest perfect square that is a factor of 8
Step 2) The value is rewritten as the product of the square root of 4 and its matching factor of 2

$$\sqrt{8} = \sqrt{4}\sqrt{2}$$

Step 3) Simplify

$$\sqrt{8} = 2\sqrt{2}$$

Simplify the following:

1) $\sqrt{75}$ 2) $\sqrt{200}$ 3) $\sqrt{108}$

4) $\sqrt{32}$ 5) $\sqrt{26}$ 6) $\sqrt{250}$

How to Simplify Radicals with Coefficients

Let's look at $3\sqrt{8}$ to help us understand the steps involving in simplifying radicals that have coefficients. All that you have to do is simplify the radical like normal and, at the end, multiply the coefficient by any numbers that 'got out' of the square root.

Step 1) Find the largest perfect square that is a factor of the radicand (just like before)	4 is the largest perfect square that is a factor of 8
Step 2) Rewrite the radical as a product of the square root of 4 (found in last step) and its matching factor(2)	$3\sqrt{4}\sqrt{2}$
Step 3) Simplify	$3\sqrt{4}\sqrt{2} = 3 \cdot 2\sqrt{2}$
Step 4) Multiply original coefficient (3) by the 'number that got out of the square root ' (2)	$3 \cdot 2\sqrt{2} = 6\sqrt{2}$

Simplify the following:

1) $2\sqrt{80}$ 2) $4\sqrt{125}$ 3) $6\sqrt{20}$

4) $3\sqrt{60}$ 5) $5\sqrt{128}$ 6) $2\sqrt{27}$