Graphical Methods in Physics – Graph Interpretation and Linearization

Part 1: Graphing Techniques

In Physics we use a variety of tools – including words, equations, and graphs – to make models of the motion of objects and the interactions between objects in a system. Graphs are one of the best ways to directly visualize the quantitative relationship between two variables – in other words, whether the variables are directly proportional, inversely proportional, not related at all, or something else entirely.

When we construct a graph, we plot the independent variable – the variable that the experimenter controls – on the x-axis, and the dependent variable – the variable that responds when the independent variable is changed – on the y-axis. There are also control variables – variables that are kept constant throughout the experiment so that they do not influence the data. So, for example, if you were trying to determine how the period of a pendulum changes when the length of the pendulum is varied, the dependent variable would be the pendulum's period, and the independent variable would be the pendulum's length. Controlled variables would include the pendulum's mass and the angle at which the pendulum was launched.

An appropriate graph for this experiment is shown below.
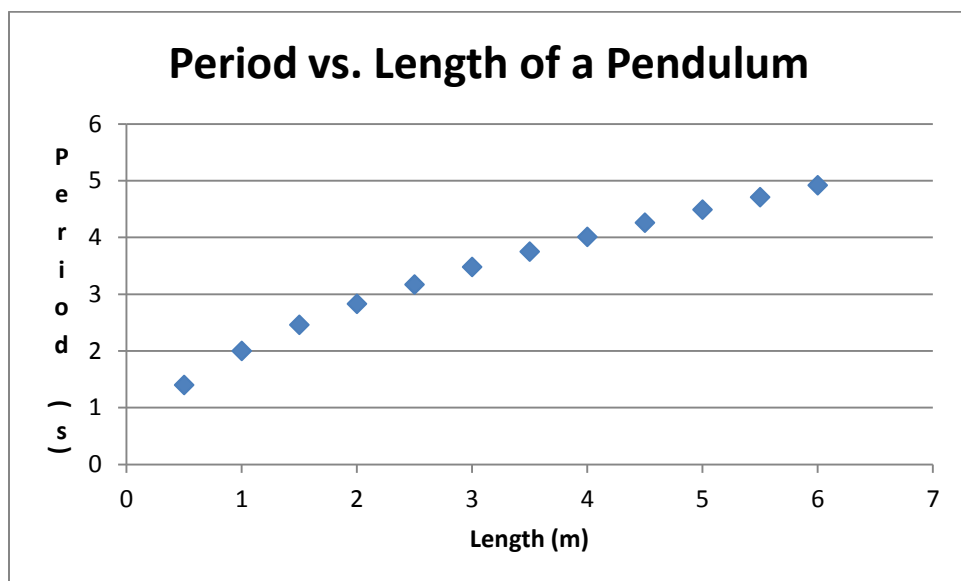
Notice that the title lists the dependent variable, which is plotted on the y-axis, first, and the independent variable, which is plotted on the x-axis, second. The axes are correctly labeled with the appropriate units. The graph begins at (0, 0) with no "jumps", and increments are equally spaced.
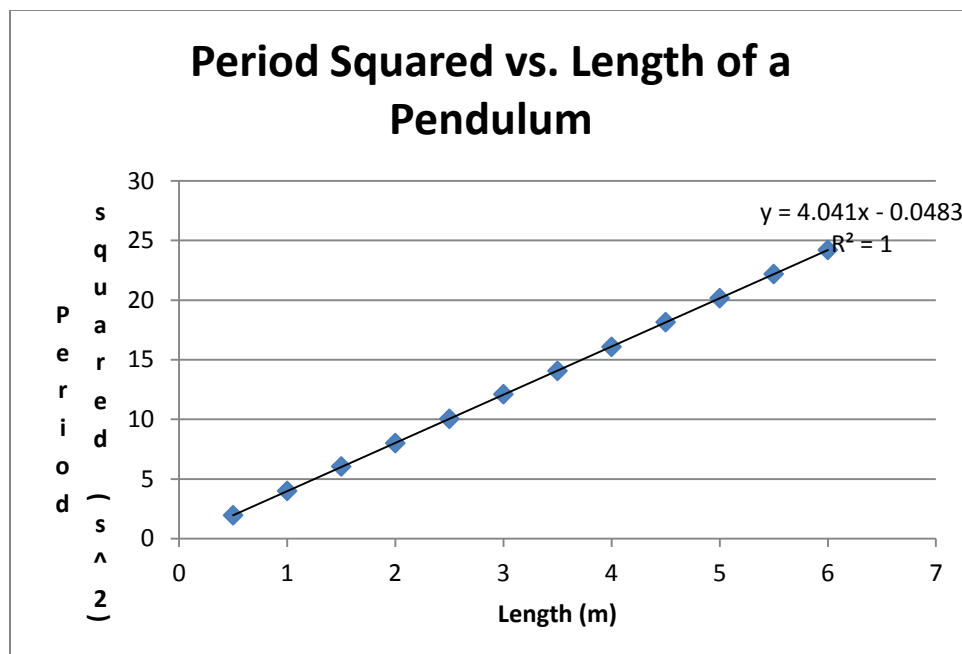
In this experiment, we can clearly see that as the length of the pendulum increases, the period also increases, but are the variables directly proportional? In other words, can we write an equation for the relationship in the form y = mx + b? Excel will draw a trend line for a graph that can help us to determine this.

Period vs. Length of a Pendulum

$y = 0.6102x + 1.4735$
$R^2 = 0.9761$

Period (s) vs. Length (m)
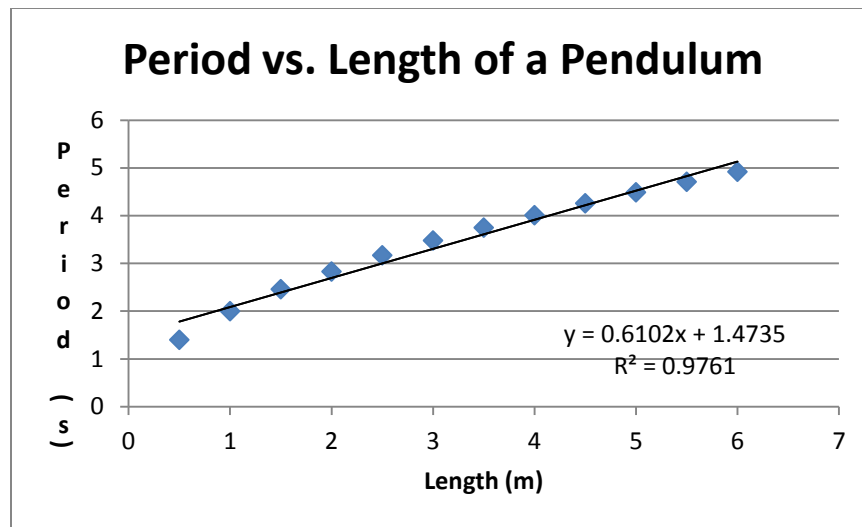
While the graph appears to be somewhat linear, we can see a few problems – first, the majority of the points do not fall on the line; second, the line does not cross the y-axis at zero, and we would expect it to – after all, a pendulum with a length very close to zero meters should have a period very close to zero seconds. To determine the correct relationship between the variables, we will have to *linearize* the graph.

Part 2: Linearization

Linearizing a graph means modifying the dependent and/or independent variables so that when you graph them, a straight line appears. The simplest way to do this is to match the shape of your graph to one of several typical shapes that you would expect to see in a Physics class. Appendix A displays the common types of graphs you will encounter in this class. Our graph appears to match the last relationship, "the square of y is proportional to x". To linearize this graph, follow the directions in the third column – in other words, plot a graph in which the square of the period is on the y-axis, and the length of the pendulum is on the x-axis. This new, linearized graph is shown below:

Period Squared vs. Length of a Pendulum

$y = 4.041x - 0.0483$
$R^2 = 1$

Period squared (s)^2 vs. Length (m)

As you can see, the modified graph displays a straight line with a y-intercept that is very close to zero, as expected. This represents a good linear fit.

Part 3: Ok, now what?

Once we have our linear fit, we can determine the mathematical relationship between the variables in our experiment. In this case, the general equation of our line is $y = 4.041x - 0.0483$. Since we know the actual variables, and the units corresponding to them, that were plotted on the x and y axes, we can substitute them into the equation:

$$T^2 = 4.041 \frac{s^2}{m} l - 0.0483 \, s^2$$

The units for the slope and y-intercept are taken directly from the graph. This equation tells us that the square of the period of a pendulum is directly proportional to its length, with the slope being a proportionality constant, $4.041 \, s^2/m$. In general, the slope has some sort of physical meaning related to the variables in the experiment that you will be asked to determine, which we will now do.

In your physics textbook, the equation that relates the period of a pendulum to its length is as follows:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

where g is the acceleration due to gravity at the surface of Earth. This equation can be written to more closely resemble the equation we determined from the graph:

$$T^2 = 4\pi^2 \frac{l}{g}$$

And it can be manipulated even further so that the equation is in the form $y = mx + b$:

$$T^2 = \frac{4\pi^2}{g} l + 0$$

Here, we can see that the slope of this line should be equal to $\frac{4\pi^2}{g}$. A typical AP Physics 1 Lab Experiment Free Response Question might ask a student to use the slope of a graph like the one above to determine a constant such as g. We can now do this by setting the equation for the slope equal to the experimental value that we determined, and then algebraically solving for the unknown constant. This is what this step would look like:

$$\frac{4\pi^2}{g} = 4.041 \frac{s^2}{m}$$

$$\frac{g}{4\pi^2} = \frac{1}{4.041} \frac{m}{s^2}$$

$$g = \frac{4\pi^2}{4.041} \frac{m}{s^2}$$

$$g = 9.77 \frac{m}{s^2}$$

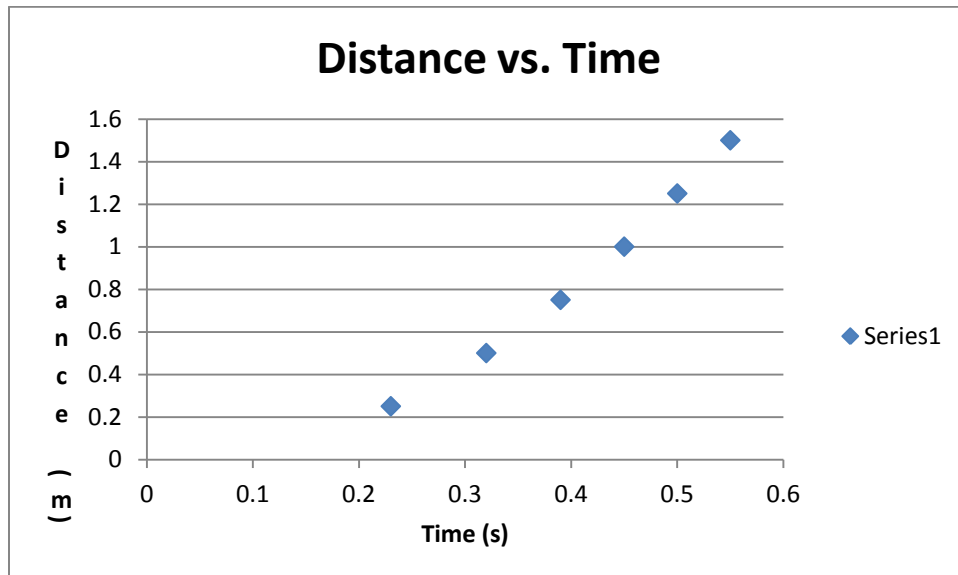This is very close to the known value of g, $9.81 \, m/s^2$.

Part 4: Your Turn!

The following graphical interpretation problem is similar to ones that appeared in prior AP Physics B Exams.

A student wishes to determine the acceleration due to gravity. She devises an apparatus that determines the time it takes for a small ball to fall when dropped from rest from different heights. The apparatus collects the following data:

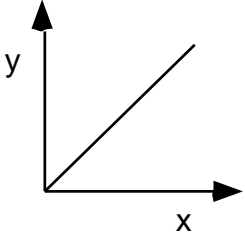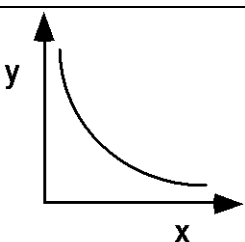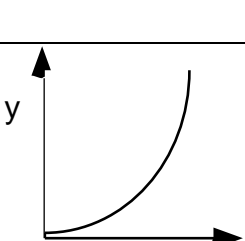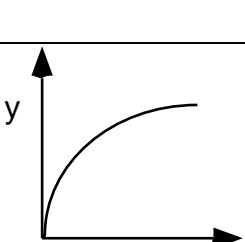| Distance (m)	Time (s)
0.25	0.23
0.50	0.32
0.75	0.39
1.00	0.45
1.25	0.50
1.50	0.55

She plots the graph shown below:
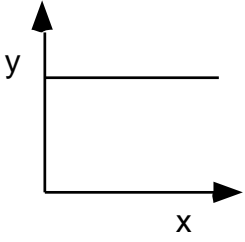
(Notice that even though the student changed the heights herself, distance is plotted as the *dependent* variable because we are trying to determine how the distance the ball traveled *depended* on the time it was in free-fall). Explain how you would linearize this data, and use Excel (see Appendix B) to create a linearized graph. Then, write an equation (with correct variables and units) relating distance to time for an object in free-fall.

If the actual equation is distance $= 1/2gt^2$, use your graph to determine the value of g.

The following graph shapes represent relationships between variables that you are likely to encounter in Physics:

Graph shape	Written relationship	Modification required to linearize graph	Algebraic representation
	As x increases, y remains the same. There is no relationship between the variables.	None	$y = b$, or y is constant
	As x increases, y increases proportionally. Y is directly proportional to x.	None	$y = mx + b$
	As x increases, y decreases. Y is inversely proportional to x.	Graph y vs $\dfrac{1}{x}$, or y vs x^{-1}	$y = m\left(\dfrac{1}{x}\right) + b$
	Y is proportional to the square of x.	Graph y vs x^2	$y = mx^2 + b$
	The square of y is proportional to x.	Graph y^2 vs x	$y^2 = mx + b$

Appendix B: Plotting a Graph in Excel

1. Open Excel. Under column "A", list your data that will be placed on the "x" axis of the graph. Under column "B", list your data that will be placed on the "y" axis of the graph.

2. Using your mouse, highlight the data.

3. Go to the "Insert" menu, and select "Scatter".

4. Choose the first box (no lines).

5. Using the "Chart Tools" menu, title your graph and label the x and y axis, with correct units.

6. Click the chart area. Go to the drop-down menu under "Chart Tools". Select "Trendline," and "More Trendline Options"

7. Choose "Linear" if you believe your graph is linear. Otherwise, modify the data as needed and create a new linear graph.