Washington Township School District

The mission of the Washington Township Public Schools *is to provide a safe, positive, and progressive educational environment that provides opportunity for all students to attain the knowledge and skills specified in the NJ Learning Standards at all grade levels, so as to ensure their full participation in an ever-changing world as responsible, self-directed and civic-minded citizens.*

Course Title:	**Advanced Placement Calculus AB**

Grade Level(s):	12

Duration:	*Full Year:*	**X**	*Semester:*		*Marking Period:*	
Course Description:	The course will cover rates of change of functions, slope, velocity, equation of the tangent and normal line, limits, and continuity; differentiation of algebraic and transcendental functions, differentials; curve sketching, related rates, Mean Value Theorem, and applications to real world both analytically and graphically. Additional topics include - Integration of simple algebraic and transcendental functions, area under a curve using Riemann sums and Trapezoidal Rule, definite integrals, Fundamental Theorem of Calculus, integration by substitution and pattern recognition; slope fields, applications to the position function, areas between curves, solids of revolution (disk, washer, and cross sectional); applications to real world accumulations both analytically and graphically. Students will practice taking AP exams from previous years to prepare for the AP Exam.					
Grading Procedures:	Each semester will be a composite of quiz scores, test scores, homework, and participation reflecting a student's mastery of the areas outlined above. The student can pass the course with an overall average of 70%. The individual teacher will explain the grading system to the student.					
Primary Resources:	NJ Student Learning Standards Mathematics (NJSLS-M) Calculus: Graphical, Numerical, Algebraic, 6th Edition Author(s): Finney, Ross | Demana, Franklin | Waits, Bert | Kennedy, Daniel MyMathLab, MyMathLab© for School APClassroom.collegeboard.org					

Washington Township Principles for Effective Teaching and Learning

- Implementing a standards-based curriculum
- Facilitating a learner-centered environment
- Using academic target language and providing comprehensible instruction
- Adapting and using age-appropriate authentic materials
- Providing performance-based assessment experiences
- Infusing 21st century skills for College and Career Readiness in a global society

Designed by:	Natalie Barretta

Under the Direction of:	Dr. Carole English

Written: 07/30/2022

Revised: _____

BOE Approval: _____

Unit Title: 1 – Limits and Continuity

Unit Description:
The concept of limit is one of the ideas that distinguishes calculus from algebra and trigonometry. Limit will be defined, and calculations completed using substitution, graphical investigation, numerical approximation, and algebra and/or some combination of these. Limits will be used to test for continuity. Continuous functions arise frequently in scientific work involving natural behaviors.

Unit Duration: 4 weeks

Desired Results

Standard(s):
N.Q.A Reason quantitatively and use units to solve problems.
F.BF.B Build new functions from existing functions.
A.SSE.A Interpret the structure of expressions.
A.APR.A Perform arithmetic operations on polynomials.
A.APR.B Understand the relationship between zeros and factors of polynomials.
A.APR.D Rewrite rational expressions.
F.IF.C Analyze functions using different representations.

Indicators:
N.Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
F.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them**.**
A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

- 1a. Interpret parts of an expression, such as terms, factors, and coefficients.
- 1b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P 2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

A.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A.APR.B.3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
A.APR.D.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
F.IF.C.8-9

- 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Understandings:
Students will understand that…

- Limits lay a foundation for calculus.
- The value of a limit means an arbitrarily close value of a function.
- There are multiple ways to evaluate a limit: graphical, numerical, and algebraic. There are two other limit scenarios frequently encountered in calculus – infinite limits and limits at infinity.
- The concept of continuity is connected to limits.

Essential Questions:

- How are limits important to calculus?
- What is the connection of limits to differentiation and integration?
- How can you read the value of a limit from a graph?
- What are the limit laws?
- How are limits evaluated?
- What is a one-sided limit?
- Why is the Squeeze Theorem useful?
- What is the difference between infinite limits and limits at infinity?
- How are vertical asymptotes related to infinite limits?
- How are horizontal asymptotes related to limits at infinity?
- What are the three conditions for continuity?
- What are the types of discontinuity?
- What is the Intermediate Value Theorem?

Assessment Evidence

Performance Tasks:

Checkpoints

Homework

MyMathLab Assignments

AP Examination Preparation

AP Free Response Preparation

Lesson Quizzes

Mid-Chapter Quiz

Exit / Admit Tickets

Other Evidence:

MyMathLab Remediation

Skill Refresher

Review and Refresh exercises

Vocabulary concept check

Study Strategies

Calculus Workbook Help

Benchmarks:
Chapter 1 Test
AP Chapter 1 Free Response Assessment

Learning Plan

Learning Activities:
*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
Guided notes
Ed Puzzle Notes – optional
Class discussions
Collaborative group work/discussions
Checkpoint/Independent practice
Exit tickets

Homework (online MyMathLab)

Unit Content :

1.1 Rates of Changes and Limits

- Interpretation and expression of limits using correct notation.
- Estimation of limits using numerical and graphical information.
- Limits of sums, differences, products, quotients, and composite functions.
- Interpretation and expression of one-sided limits.
- The Squeeze Theorem.

1.2 Limits Involving Infinity

- The Squeeze Theorem for limits at infinity.
- Asymptotic and unbounded behavior of functions.
- End behavior of functions.

1.3 Continuity

- Definition of continuity at a point.
- Types of discontinuities.
- Sums, differences, products, quotients, and compositions of continuous functions.
- Common continuous functions.
- Continuity and the Intermediate Value Theorem.

1.4 Rates of Change, Tangent Lines, and Sensitivity

- Use limits to determine instantaneous rats of change, slopes of tangent lines, and sensitivity to change.

Chapter Review

Chapter Test

Resources:

MyMathLab online textbook and practice

QR codes in textbook for access to instructional videos, solutions to exercise and Checkpoint exercises.

MathXL.com for video solutions of selected exercises.

Unit Modifications for Special Population Students	
Advanced Learners	• Invite students to explore different points of view on a topic of study and compare the two. • Assign a leadership role in classroom learning • Determine where student's interests lie and capitalize on their inquisitiveness. • Expose students to a selection and use of specialized resources
Struggling Learners	• Be flexible with time frames and deadlines • Create planned opportunities for interaction between individuals in the classroom: cooperative and collaborative learning, pair and share with peers • Group students • Intentional scheduling/grouping with student/teacher of alternative background • Provide support as at-risk students move through all levels of knowledge acquisition • Tap prior knowledge
English Language Learners	• Accommodate with completed study guides to assist with preparation on tests • Allow students to give responses in a form (oral or written) that's easier for him/her • Be flexible with time frames, deadlines, or modify assessments • Create planned opportunities for interaction between individuals in the classroom: skits, cooperative and collaborative learning, student generated stories based on personal experience • Establish a framework allowing ELL students to understand and assimilate new ideas and information • Focus on domain specific vocabulary and keywords • Give alternate or paper copies to accommodate electronic assignments. • Have another student share class notes with the ELL student. • Intentional scheduling/grouping with student/teacher of language if possible • Mark texts with a highlighter. • Take more time to complete a task, project, or test. • Use manipulatives, graphic organizer, and real objects when possible • Use visual presentations/verbal materials (ex: word webs and visual organizers).

Special Needs Learners	• Accommodate with completed study guides to assist with preparation on tests. • Allow more time to complete task, project, or test • Allow students to give responses in a form (oral or written) that's easier for him • Be flexible with time frames, deadlines, or modify assessments • Give alternate or paper copies to replace electronic assignments • Have another student share class notes with the special needs learner. • Higher level reasoning and questioning would have less weight than other assignments. • Receive study skill instructions. • Work with fewer items per page or line and/or materials in a larger print.
Learners with a 504	Refer to page four in the Parent and Educator Resource Guide to Section 504 to assist in the development of appropriate plans.

Interdisciplinary Connections

Indicators:

ELA

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
WHST.11-12.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Computer Science and Design Thinking

8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

Technology Education

9.3.ST.1: Use technology to acquire, manipulate, analyze, and report data.
9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.

Life Literacies & Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
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9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

Integration of 21st Century Skills

Indicators:
From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration, communication, and critical thinking.

Unit Title: 2 Derivatives

Unit Description: Now that limits have been studied, the door to calculus stands open. First is the concept and definition of a derivative. The derivative gives the slope of the curve as it changes with respect to x (the tangent line). The derivative gives the instantaneous rate of change of the function *f* with respect to the independent variable. We use limits not only to define the derivative, but also to develop the rules for finding the derivative. The Chain Rule is the most widely used differentiation rule in mathematics. Implicit differentiation allows the student to find derivatives of functions that can't be explicitly defined. The applications of derivatives are numerous and are introduced throughout the unit since change is all around us and derivatives describe change.

Unit Duration: 4 weeks

Desired Results

Standard(s):
F.BF.A Build a function that models a relationship between two quantities.
F.BF.B Build new functions from existing functions.
A.SSE.A Interpret the structure of expressions.
A.SSE.B Write expressions in equivalent forms to solve problems.
A.APR.A Perform arithmetic operations on polynomials.
A.APR.B Understand the relationship between zeros and factors of polynomials.
A.APR.D Rewrite rational expressions.
F.IF.C Analyze functions using different representations.

Indicators:
F.BF.A.1 Determine an explicit expression, a recursive process, or steps for calculation from a context.
F.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them**.**
A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

- 1a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P
- 2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

A.SSE.B.3-4 3.Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. 4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems.
A.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A.APR.B.3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
A.APR.D.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
F.IF.C.8-9

- 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Understandings:	Essential Questions:
Students will understand that… • There is a difference between the average rate of change and the instantaneous rate of change. • The instantaneous rate of change is the slope of the tangent line. • The definition of the derivative is a limit function. • A graph can be drawn from the tangent lines of the function. • Students draw the derivative graph from the function graph. • There is a connection between continuity and differentiability • There are points where the derivative will fail to exist. • There is a power rule for derivatives. • There are properties for derivatives. • There are rules for finding the derivative of the product and quotient of functions. • There are rules for finding the derivatives of trigonometric functions. • The derivative can be used to find the rate of change. • There are rules for finding the derivative of composite functions. • There are some functions where the derivative cannot be found explicitly. • There are derivatives of exponential and logarithmic functions. • There are derivatives of inverse functions including inverse trigonometric functions. • Derivatives can be used to solve rates of change problems that are given with respect to time.	• What is the difference between the average rate of change and the instantaneous rate of change? • How do you find the slope of the tangent line? • How do you use the limit definition to find a derivative? • How do you draw the derivative graph from the function graph? • Is a continuous function differentiable? • Where does a derivative fail to exist? • How do you derive the power rule for derivatives? • What is the derivative of e? • How can you derive the rule for the derivative of the product and quotient of functions? • How can you derive the rules for the derivatives of trigonometric functions? • What is the difference between average and instantaneous velocity? • How is the derivative used to show motion? • How do you use the chain rule to find the derivative of composite functions? • How do you use the chain rule to find the derivative implicitly? • Using knowledge of derivatives and exponential functions develop the rules for general exponential functions. • Using knowledge of derivatives and logarithmic functions develop the rules for logarithmic functions. • What is an inverse function? • How are the derivatives related in inverse functions? • What method of differentiation must be used to solve related rate problems? • What strategies are used to solve related rate problems?

Assessment Evidence	
Performance Tasks:	**Other Evidence:**
Checkpoints	MyMathLab Remediation
Homework	Skill Refresher
MyMathLab Assignments	Review and Refresh exercises
AP Examination Preparation	Vocabulary concept check
AP Free Response Preparation	Study Strategies
Lesson Quizzes	Calculus Workbook Help

Mid-Chapter Quiz Exit / Admit Tickets	

Benchmarks:
Chapter 2 Assessment
AP Chapter 2 Free Response Assessment

Learning Plan

Learning Activities:
*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
Guided notes
Ed Puzzle Notes – optional
Class discussions
Collaborative group work/discussions
Checkpoint/Independent practice
Exit tickets
Homework (online MyMathLab)

Unit Content :

2.1 Derivative of a Function

- Compute the derivative of a function at x = a using both forms of the limit definition and explain its relationship to slope.
- Identify different ways of denoting the derivative of a function.
- One-sided derivatives.
- Graphing the derivative from data.
- Graphing y = f(x) given the graph of y = f'(x).
- Graphing y = f'(x) given the graph of y = f(x).

2.2 Differentiability

- Analyze and discuss the differentiability of functions.
- Understand why f'(a) might fail to exist at f=a.
- Use the Intermediate Value Theorem for derivatives.
- Find numerical derivatives on a calculator.

2.3 Rules for Differentiation

Apply the rules for differentiation using…

- Power functions
- Sum and difference rules
- Product and quotient rules
- Negative integer powers of x
- Second and higher order derivatives

2.4 Velocity and Other Rates of Change

- Interpret the derivative as representing velocity and other rates of change.
- Instantaneous rates of change.
- Motion on a line.
- Acceleration as the second derivative.
- Modeling vertical motion and particle motion.
- Derivative as a measure of sensitivity to change.
- Marginal cost and marginal revenue.

2.5 Derivative of Trigonometric Functions

- Determine the derivatives of trigonometric functions.
- Find derivatives of the sine and cosine function.
- Model harmonic motion.
- Find derivatives of the tangent, cotangent, secant, and secant functions.
- Find tangent lines and normal lines.

Chapter Review

Chapter Test

Resources:

MyMathLab online textbook and practice

QR codes in textbook for access to instructional videos, solutions to exercise and Checkpoint exercises.

MyMathLab for video solutions of selected exercises.

Unit Modifications for Special Population Students	
Advanced Learners	• Invite students to explore different points of view on a topic of study and compare the two. • Assign a leadership role in classroom learning • Determine where student's interests lie and capitalize on their inquisitiveness • Expose students to a selection and use of specialized resources
Struggling Learners	• Be flexible with time frames and deadlines • Create planned opportunities for interaction between individuals in the classroom: cooperative and collaborative learning, pair and share with peers • Group students • Intentional scheduling/grouping with student/teacher of alternative background • Provide support as at-risk students move through all levels of knowledge acquisition • Tap prior knowledge
English Language Learners	• Accommodate with completed study guides to assist with preparation on tests • Allow students to give responses in a form (oral or written) that's easier for him/her • Be flexible with time frames, deadlines, or modify assessments • Create planned opportunities for interaction between individuals in the classroom: skits, cooperative and collaborative learning, student generated stories based on personal experience • Establish a framework allowing ELL students to understand and assimilate new ideas and information • Focus on domain specific vocabulary and keywords • Give alternate or paper copies to accommodate electronic assignments. • Have another student share class notes with the ELL student. • Intentional scheduling/grouping with student/teacher of language if possible • Mark texts with a highlighter. • Take more time to complete a task, project, or test. • Use manipulatives, graphic organizer, and real objects when possible • Use visual presentations/verbal materials (ex: word webs and visual organizers).

Special Needs Learners	• Accommodate with completed study guides to assist with preparation on tests. • Allow more time to complete task, project, or test • Allow students to give responses in a form (oral or written) that's easier for him • Be flexible with time frames, deadlines, or modify assessments • Give alternate or paper copies to replace electronic assignments • Have another student share class notes with the special needs learner. • Higher level reasoning and questioning would have less weight than other assignments. • Receive study skill instructions. • Work with fewer items per page or line and/or materials in a larger print.
Learners with a 504	Refer to page four in the Parent and Educator Resource Guide to Section 504 to assist in the development of appropriate plans.

Interdisciplinary Connections

Indicators:

ELA

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
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Computer Science and Design Thinking

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9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
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Integration of 21st Century Skills

Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration and communication.

Unit Title: 3 More Derivatives

Unit Description: Now that students have completed a chapter filled with derivative rules and their proofs (most of which teachers hope students were able to understand rather than simply memorize), students are about to embark on another derivative chapter, this one devoted to a single rule: the Chain Rule. This rule certainly deserves its own chapter, but teachers also want students to become sufficiently familiar with the rich variety of applications that are accessible once they know how to apply it.

Unit Duration: 3 weeks

Desired Results

Standard(s):

F.BF.A Build a function that models a relationship between two quantities.
F.BF.B Build new functions from existing functions.
A.SSE.A Interpret the structure of expressions.
A.SSE.B Write expressions in equivalent forms to solve problems.
A.APR.A Perform arithmetic operations on polynomials.
A.APR.B Understand the relationship between zeros and factors of polynomials.
A.APR.D Rewrite rational expressions.
F.IF.C Analyze functions using different representations.

Indicators:

F.BF.A.1 Determine an explicit expression, a recursive process, or steps for calculation from a context.
F.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them**.**
A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

- 1a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P
- 2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

A.SSE.B.3-4 3.Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. 4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems.
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A.APR.D.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
F.IF.C.8-9

- 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Understandings:

Students will understand that…

- The Chain Rule is used for differentiating a composite function.
- The Chain Rule in prime and Leibniz notations.
- There is a Chain Rule to show how degree measure affects the calculus of trig functions.
- There are implicitly defined functions.
- How to use the Chain Rule to find derivatives of functions defined implicitly.
- There are tangent and normal lines to implicitly defined curves.
- There are higher-order derivatives of implicitly defined functions.
- There are extensions to the Power Rule from integer powers to rational powers.
- You can use implicit differentiation to find the derivatives of inverses of functions.
- There are graphical interpretations of how the derivate of the function *f* is related to the derivative of f^{-1}.
- There are derivatives of inverse trigonometric functions.
- There are identities for graphing arccot, arcsec, and arccsc functions.
- There are derivatives of e^x, a^x, lnx, and $log_a x$.
- There are multiple methods to extend the Power Rule to arbitrary real powers.

Essential Questions:

- What is the Chain Rule?
- How do you use the Chain Rule to show how degree measure affects the calculus of trig functions?
- How do you differentiate parametrically defined functions?
- How do you find the derivative of an implicit function?
- How do you use the chain rule to find derivatives of functions defined implicitly?
- How do you extend the Power Rule from integer powers to rational powers?
- What is a higher-order derivative?
- What does it mean to differentiate inverse functions?
- What does the graphical interpretation of how the derivative of the function *f* is related to the derivative of its inverse look like?
- How do you graph the identities of arcsec, arccot, and arccsc on the graphing calculator?
- What is the derivative of e^x?
- What is the derivative of a^x?
- What is the derivative of lnx?
- What is the derivative of $log_a x$?
- How do you use logarithmic differentiation to find derivatives?

Assessment Evidence

Performance Tasks:

Checkpoints

Homework

MyMathLab Assignments

AP Examination Preparation

AP Free Response Preparation

Lesson Quizzes

Mid-Chapter Quiz

Exit / Admit Tickets

Other Evidence:

MyMathLab Remediation

Skill Refresher

Review and Refresh exercises

Vocabulary concept check

Study Strategies

Calculus Workbook Help

Benchmarks:
Chapter 3 Assessment
AP Chapter 3 Free Response Assessment

Learning Plan

Learning Activities:
*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
Guided notes
Ed Puzzle Notes – optional
Class discussions
Collaborative group work/discussions
Checkpoint/Independent practice
Exit tickets
Homework (online MyMathLab)

Unit Content:
3.1 Chain Rule
- Use the chain rule for differentiating a composite function.
- Use the chain rule in prime and Leibniz notations.
- Differentiate parametrically defined functions.
- Use the power chain rule.
- Use the chain rule to show how degree measure affects the calculus of trig functions.

3.2 Implicit Differentiation
- Implicitly defined functions.
- Using the chain rule to find derivatives of functions defined implicitly.
- Tangent and normal lines to implicitly defined curves.
- Finding higher order derivatives of implicitly defined functions.
- Extending the power rule from integer powers to rational powers.

3.3 Derivatives of Inverse Trigonometric Functions
- Differentiate inverse functions.
- Understand the graphical interpretation of how the derivative of the function *f* is related to the derivative of f^{-1}
- Derivatives of the inverse trigonometric functions.
- Identities for graphing arccot, acrsec, arccsc functions on a graphing calculator.

3.4 Derivatives of Exponential and Logarithmic Functions
Find derivatives of exponential functions and logarithmic functions with positive base a ($a \neq 1$).
- Derivative of e^x.
- Derivative of a^x.
- Derivative of lnx.
- Derivative of $log_a x$.
- Extending the Power Rule to arbitrary real powers.

- Logarithmic differentiation.

Chapter Review

Chapter Test

Resources:

MyMathLab online textbook and practice

QR codes in textbook for access to instructional videos, solutions to exercise and Checkpoint exercises.

MyMathLab for video solutions of selected exercises.

Unit Modifications for Special Population Students	
Advanced Learners	• Invite students to explore different points of view on a topic of study and compare the two. • Assign a leadership role in classroom learning • Determine where student's interests lie and capitalize on their inquisitiveness • Expose students to a selection and use of specialized resources
Struggling Learners	• Be flexible with time frames and deadlines • Create planned opportunities for interaction between individuals in the classroom: cooperative and collaborative learning, pair and share with peers • Group students • Intentional scheduling/grouping with student/teacher of alternative background • Provide support as at-risk students move through all levels of knowledge acquisition • Tap prior knowledge
English Language Learners	• Accommodate with completed study guides to assist with preparation on tests • Allow students to give responses in a form (oral or written) that's easier for him/her • Be flexible with time frames, deadlines, or modify assessments • Create planned opportunities for interaction between individuals in the classroom: skits, cooperative and collaborative learning, student generated stories based on personal experience • Establish a framework allowing ELL students to understand and assimilate new ideas and information • Focus on domain specific vocabulary and keywords • Give alternate or paper copies to accommodate electronic assignments. • Have another student share class notes with the ELL student. • Intentional scheduling/grouping with student/teacher of language if possible • Mark texts with a highlighter. • Take more time to complete a task, project, or test. • Use manipulatives, graphic organizer, and real objects when possible • Use visual presentations/verbal materials (ex: word webs and visual organizers).

Special Needs Learners	• Accommodate with completed study guides to assist with preparation on tests. • Allow more time to complete task, project, or test • Allow students to give responses in a form (oral or written) that's easier for him • Be flexible with time frames, deadlines, or modify assessments • Give alternate or paper copies to replace electronic assignments • Have another student share class notes with the special needs learner. • Higher level reasoning and questioning would have less weight than other assignments. • Receive study skill instructions. • Work with fewer items per page or line and/or materials in a larger print.
Learners with a 504	Refer to page four in the Parent and Educator Resource Guide to Section 504 to assist in the development of appropriate plans.

Interdisciplinary Connections

Indicators:

ELA

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
WHST.11-12.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Computer Science and Design Thinking

8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

Technology Education

9.3.ST.1: Use technology to acquire, manipulate, analyze, and report data.
9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.

Life Literacies & Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
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Integration of 21st Century Skills

Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration and communication.

Unit Title: 4 Applications of Derivatives

Unit Description: Much of the previous unit explored the basic mechanics of derivatives: evaluating them and interpreting them as rates of change. It is now time to apply derivatives to a variety of mathematical questions including applying derivatives to functions and their graphs. Determining the minimum and maximum points of graphs using derivatives and using these points to answer real world questions. First and second derivatives will be used to justify answers. In this unit the discussion begins about approximating functions and the Mean Value Theorem for Derivatives, which will connect the average and instantaneous rates of change.

Unit Duration: 4 weeks

Desired Results

Standard(s):

N.Q.A Reason quantitatively and use units to solve problems.
G.C.A Understand and apply theorems about circles.
F.BF.A Build a function that models a relationship between two quantities
A.SSE.A Interpret expressions that represent a quantity in terms of its context.
A.SSE.B Write expressions in equivalent forms to solve problems
A.CED.A Create equations that describe numbers or relationships
A.REI.B Solve equations and inequalities in one variable
A.APR.A Perform arithmetic operations on polynomials
A.APR.B Understand the relationship between zeros and factors of polynomials
A.APR.D Rewrite rational expressions
F.IF.B Interpret functions that arise in applications in terms of the context
F.LE. A Construct and compare linear and exponential models and solve problems
F.LE.B Interpret expressions for functions in terms of the situation they model

Indicators:

N.Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
G.C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
F.BF.A.1 Determine an explicit expression, a recursive process, or steps for calculation from a context.
A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.1 a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P 2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
A.SSE.B.3-4 3.Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. 4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems
A.CED.A.1-4

- 1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- 2 Create equations in two or more variables to represent relationships between quantities, graph equations on coordinate axes with labels and scales.
- 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
- 4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

A.REI.B.3-4a

- 3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- 4. Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form.

A.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A.APR.B.3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
A.APR.D.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
F.IF.B 4-6

- 4.For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
- 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.LE.A.1-3

- 1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
- 2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- 3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

Understandings: *Students will understand that…*	**Essential Questions:**
• Every closed function has a minimum and a maximum point • The first and second derivatives tell us features of the function. • A function can be graphed using the first and second derivatives. • An optimization problem can be solved by finding the extrema of an objective function. • Functions can be approximated using linear approximation and differentials. • The Mean Value Theorem connects average and instantaneous rates of change. • L'Hopital's Rule is another way to evaluate limits in an indeterminate form. • Roots to a function can be found using Newton's Method.	• How do you determine if an extrema is a maximum or a minimum? • What is the difference between an absolute and local minimum and maximum? • What is a critical point? • When is a function increasing or decreasing? • What is the test for an increasing or decreasing function? • What is the First Derivative Test? • When is a function concave up or concave down? • What is the Second Derivative Test? • What are the guidelines for graphing a function? • What else is useful besides the derivatives? • Given the graph of the first and second derivatives, can the graph of the function be drawn? • What is an objective function? • What are the guidelines for optimization problems? • How do you justify your answer in an optimization problem?

	• What is the formula for linearization? • What is a differential? • What is the physical interpretation of the Mean Value Theorem? • What is Rolle's Theorem? • What is an indeterminate form? • What is L'Hopital's Rule? • What are growth rates of functions? • What is Newton's Method for approximating functions?

Assessment Evidence

Performance Tasks:	**Other Evidence:**
Checkpoints	MyMathLab Remediation
Homework	Skill Refresher
MyMathLab Assignments	Review and Refresh exercises
AP Examination Preparation	Vocabulary concept check
AP Free Response Preparation	Study Strategies
Lesson Quizzes	Calculus Workbook Help
Mid-Chapter Quiz	
Exit / Admit Tickets	

Benchmarks:
Chapter 4 Assessment
AP Chapter 4 Free Response Assessment

Learning Plan

Learning Activities:
*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
Guided notes
Ed Puzzle Notes – optional
Class discussions
Collaborative group work/discussions
Checkpoint/Independent practice
Exit tickets
Homework (online MyMathLab)

Unit Content:
4.1 Extreme Value of Functions

- Find the maximum and minimum value of a function over a given interval and determine the applicability of the Extreme Value Theorem.
- Use derivatives to find extrema.
- Use The Candidates Test for absolute extrema.

4.2 Mean Value Theorem
- Apply the Mean Value Theorem to describe behavior of a function over an interval.
- Use the Mean Value Theorem to determine increasing and decreasing functions.

4.3 Connecting f' and f'' with the Graph of f.
- Use derivatives to analyze properties of functions.
- Use the first derivative test for local extrema.
- Use the concavity test to determine the concavity of functions.
- Use the second derivative test to find extrema.

4.4 Modeling and Optimization
- Use derivatives to solve optimization problems.
- Analyze a function used to model a situation.
- Develop a mathematical model and interpret the solution.
- Model discrete phenomena with differentiable functions.

4.5 Linearization, Sensitivity, and Differentials
- Solve problems involving the slope of the tangent line.
- Find and use linearization.
- Find the differential and evaluate for given values.
- Use Newton's Method to solve problems.

4.6 Related Rate Equations
- Solve problems involving rates of change in applied contexts.
- Develop a mathematical model.
- Create an equation that relates the variable whose rate of change is known to the variable whose rate of change is sought.
- Use the Chain Rule to relate the rates of change.
- Identify and interpret a solution.

Chapter Review

Chapter Test

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Interdisciplinary Connections

Indicators:

ELA

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Life Literacies & Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).

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Integration of 21st Century Skills

Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration and critical thinking.

Unit Title: 5 The Definite Integral

Unit Description:
This unit connects the two branches of calculus: derivatives and integrals. It begins by undoing derivatives – antidifferentiation. Next is finding area under the curve by accumulating the area of rectangles. The Fundamental Theorem of Calculus will bring differential and integral calculus together to become the single most powerful insight mathematicians had ever acquired for understanding how the universe works.

Unit Duration: 5 weeks

Desired Results

Standard(s):
F.BF.A Build a function that models a relationship between two quantities.
F.BF.B Build new functions from existing functions.
A.SSE.A Interpret the structure of expressions.
A.SSE.B Write expressions in equivalent forms to solve problems.
A.APR.A Perform arithmetic operations on polynomials.
A.APR.B Understand the relationship between zeros and factors of polynomials.
A.APR.D Rewrite rational expressions.
F.IF.C Analyze functions using different representations.

Indicators:
F.BF.A.1 Determine an explicit expression, a recursive process, or steps for calculation from a context.
F.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them**.**
A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

- 1a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P
- 2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

A.SSE.B.3-4 3.Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. 4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems.
A.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A.APR.B.3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
A.APR.D.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
F.IF.C.8-9

- 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Understandings:
Students will understand that…

- Antiderivatives are the reverse process to differentiation and differ by a constant of integration.
- Using the sum of the areas of a series of rectangles will approximate the area under a curve.
- The net area is the same as the displacement and the total area is the area between the curve and the x-axis.
- The Fundamental Theorem of Calculus is the single most powerful concept in mathematics, connecting derivatives and integrals.
- There is exactly one rectangle that will approximate the area under the curve.
- More difficult integration problems can be solved using integration by substitution.
- Another way to approximate the area under the curve is to use the trapezoid rule.

Essential Questions:

- What is an antiderivative?
- What is the power rule for integrals?
- What are the rules for integrals?
- What are the indefinite integrals for trigonometric functions?
- What is a differential equation?
- Which Rectangular Approximation method is the most accurate?
- What is Riemann Sum?
- What is a definite integral?
- What is the difference between the net and total areas?
- What are the properties of integrals?
- What is the Fundamental Theorem of Calculus?
- How does the Fundamental Theorem of Calculus connect derivatives and integrals?
- What is the average value of a function?
- What is the Mean Value Theorem for integrals?
- How is integration by substitution used to find the integrals of composite functions?
- What are the integrals of the rest of the trigonometric functions?
- What is the trapezoid rule?

Assessment Evidence

Performance Tasks:

Checkpoints

Homework

MyMathLab Assignments

AP Examination Preparation

AP Free Response Preparation

Lesson Quizzes

Mid-Chapter Quiz

Exit / Admit Tickets

Other Evidence:

MyMathLab Remediation

Skill Refresher

Review and Refresh exercises

Vocabulary concept check

Study Strategies

Calculus Workbook Help

Benchmarks:
Chapter 5 Assessment
AP Chapter 5 Free Response Assessment

Learning Plan

Learning Activities:
*Lessons may include some or all of the following activities
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Review of Homework
Guided notes
Ed Puzzle Notes – optional
Class discussions
Collaborative group work/discussions
Checkpoint/Independent practice
Exit tickets
Homework (online MyMathLab)

Unit Content:

5.1 Estimating with Finite Sums

- Estimate distance, areas, volumes, and accumulations using finite sums.
- Estimate accumulation as area under the curve representing rate of accumulation.
- Estimate area using rectangular approximation.
- Use of left, right, and midpoint sums to approximate areas.

5.2 Definite Integrals

- Interpret the definite integral as the limit of a Riemann sum and express the limit.
- Understand the existence of definite integrals for continuous functions.
- Integrate a discontinuous function.

5.3 Definite Integrals and Antiderivatives

- Calculate a definite integral using areas and properties of definite integrals.
- Apply definite integrals to problems involving definite integrals.
- Use the Mean Value Theorem for definite integrals.

5.4 Fundamental Theorem of Calculus

- Analyze functions defined by an integral and evaluate definite integrals.
- Apply the Fundamental Theorem of Calculus.
- Evaluate a definite integral using antiderivatives.
- Use of definite integrals to define new functions.

5.5 Trapezoidal Sums

- Approximate definite integrals using trapezoidal sums.
- Compare trapezoidal approximations to other numerical approximations.

Chapter Review

Chapter Test

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Integration of 21st Century Skills

Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration and communication.

Unit Title: 6 Differential Equations and Mathematical Modeling

Unit Description:
Differential equations lie at the heart of mathematical modeling and are used in engineering, physics, chemistry, biology geophysics, economics, and finance. It has many applications including analyzing the stability of buildings, simulating planet and satellite orbits, modeling populations and epidemics. Differential equations rely heavily on calculus. This is just a brief discussion of differential equations.

Unit Duration: 3 weeks

Desired Results

Standard(s):
F.BF.A Build a function that models a relationship between two quantities.
F.BF.B Build new functions from existing functions.
A.SSE.A Interpret the structure of expressions.
A.SSE.B Write expressions in equivalent forms to solve problems.
A.APR.A Perform arithmetic operations on polynomials.
A.APR.B Understand the relationship between zeros and factors of polynomials.
A.APR.D Rewrite rational expressions.
F.IF.C Analyze functions using different representations.

Indicators:
F.BF.A.1 Determine an explicit expression, a recursive process, or steps for calculation from a context.
F.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them**.**
A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

- 1a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P
- 2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

A.SSE.B.3-4 3.Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. 4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems.
A.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A.APR.B.3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
A.APR.D.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
F.IF.C.8-9

- 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Understandings: *Students will understand that…* • A general solution can be found for basic differential equations. • A slope field can be drawn from a differential equation that will represent the function and that students can use Euler's Method to find the value of a function at a point. • Separable differential equations is a way to solve a differential equation. • Separable differential equations can be used to solve exponential growth and decay problems.	**Essential Questions:** • What is the process of solving linear differential equations? • What are initial value problems? • What is the connection between a slope field and a differential equation? • How is the differential equation used to graph a slope field? • How is Euler's Method used to solve a differential equation? • What is a separable differential equation? • How do you solve a separable differential equation? • How are logistic equations solved using differential equations? • How are differential equations used to solve exponential growth and decay problems? • How are separable differential equations used to solve exponential growth and decay problems?
Assessment Evidence	
Performance Tasks: Checkpoints Homework MyMathLab Assignments AP Examination Preparation AP Free Response Preparation Lesson Quizzes Mid-Chapter Quiz Exit / Admit Tickets	**Other Evidence:** MyMathLab Remediation Skill Refresher Review and Refresh exercises Vocabulary concept check Study Strategies Calculus Workbook Help
Benchmarks: Chapter 6 Assessment AP Chapter 6 Free Response Assessment	
Learning Plan	

Learning Activities:
*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
Guided notes
Ed Puzzle Notes – optional
Class discussions
Collaborative group work/discussions
Checkpoint/Independent practice
Exit tickets
Homework (online MyMathLab)

Unit Content:

6.1 Slope Field and Euler's Method

- Use slope fields to analyze solution curves to differential equations.
- Use Euler's Method to construct solutions numerically.

6.2 Antidifferentiation by Substitution

- Find antiderivatives using the technique of substitution to reverse the effect of the Chain Rule.
- Evaluate an indefinite integral.
- Use substitution to evaluate indefinite integrals.

6.3 Antidifferentiation by Parts – BC TOPIC (COVER IF TIME ALLOWS)

- Find antiderivatives of functions using the technique of parts.
- Derive the parts formula from the Product Rule.
- Use repeated application of the parts formula.
- Solve for the unknown integral.

6.4 Exponential Growth and Decay

- Solve separable differential equations, including those arising in problems of exponential growth and decay.
- Solve continuously compounded interest problems.

Chapter Review

Chapter Test

Resources:

MyMathLab online textbook and practice

QR codes in textbook for access to instructional videos, solutions to exercise and Checkpoint exercises.

MathXL.com for video solutions of selected exercises.

Unit Modifications for Special Population Students	
Advanced Learners	• Invite students to explore different points of view on a topic of study and compare the two. • Assign a leadership role in classroom learning • Determine where student's interests lie and capitalize on their inquisitiveness • Expose students to a selection and use of specialized resources
Struggling Learners	• Be flexible with time frames and deadlines • Create planned opportunities for interaction between individuals in the classroom: cooperative and collaborative learning, pair and share with peers • Group students • Intentional scheduling/grouping with student/teacher of alternative background • Provide support as at-risk students move through all levels of knowledge acquisition • Tap prior knowledge
English Language Learners	• Accommodate with completed study guides to assist with preparation on tests • Allow students to give responses in a form (oral or written) that's easier for him/her • Be flexible with time frames, deadlines, or modify assessments • Create planned opportunities for interaction between individuals in the classroom: skits, cooperative and collaborative learning, student generated stories based on personal experience • Establish a framework allowing ELL students to understand and assimilate new ideas and information • Focus on domain specific vocabulary and keywords • Give alternate or paper copies to accommodate electronic assignments. • Have another student share class notes with the ELL student. • Intentional scheduling/grouping with student/teacher of language if possible • Mark texts with a highlighter. • Take more time to complete a task, project, or test. • Use manipulatives, graphic organizer, and real objects when possible • Use visual presentations/verbal materials (ex: word webs and visual organizers).

Special Needs Learners	• Accommodate with completed study guides to assist with preparation on tests. • Allow more time to complete task, project, or test • Allow students to give responses in a form (oral or written) that's easier for him • Be flexible with time frames, deadlines, or modify assessments • Give alternate or paper copies to replace electronic assignments • Have another student share class notes with the special needs learner. • Higher level reasoning and questioning would have less weight than other assignments. • Receive study skill instructions. • Work with fewer items per page or line and/or materials in a larger print.
Learners with a 504	Refer to page four in the Parent and Educator Resource Guide to Section 504 to assist in the development of appropriate plans.

Interdisciplinary Connections

Indicators:

ELA

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
WHST.11-12.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Computer Science and Design Thinking

8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

Technology Education

9.3.ST.1: Use technology to acquire, manipulate, analyze, and report data.
9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.

Life Literacies & Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

Integration of 21st Century Skills

Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration, communication, and critical thinking.

Unit Title: 7 Applications of Definite Integrals

Unit Description:
This unit focuses on the uses of integration. If the rate of change is known, then integration can be used to determine the net change or the future value of that quantity over a certain time interval. There are also geometric applications of integration computing the area of regions bounded by several curves, the volume, or three-dimensional solids and the length of curves.

Unit Duration: 3 weeks

Desired Results

Standard(s):
N.Q.A Reason quantitatively and use units to solve problems.
G.C.A Understand and apply theorems about circles.
F.BF.A Build a function that models a relationship between two quantities
A.SSE.A Interpret expressions that represent a quantity in terms of its context.
A.SSE.B Write expressions in equivalent forms to solve problems
A.CED.A Create equations that describe numbers or relationships
A.REI.B Solve equations and inequalities in one variable
A.APR.A Perform arithmetic operations on polynomials
A.APR.B Understand the relationship between zeros and factors of polynomials
A.APR.D Rewrite rational expressions
F.IF.B Interpret functions that arise in applications in terms of the context
F.LE.B Interpret expressions for functions in terms of the situation they model

Indicators:
N.Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
G.C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
F.BF.A.1 Determine an explicit expression, a recursive process, or steps for calculation from a context.
A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.1 a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P 2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
A.SSE.B.3-4 3.Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. 4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems
A.CED.A.1-4

- 1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- 2 Create equations in two or more variables to represent relationships between quantities, graph equations on coordinate axes with labels and scales.
- 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
- 4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

A.REI.B.3-4a

- 3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- 4. Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form.

A.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A.APR.B.3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
A.APR.D.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
F.IF.B 4-6

- 4.For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
- 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

Understandings: *Students will understand that…*	**Essential Questions:**
• There is a relationship between velocity, net change and total distance traveled and that this relationship can carry over to more general situations. • It is possible to find the area between two curves by generalizing the method of finding the area under a curve. • The volume of a solid can be found by taking the methods used for finding the area and expanding them to three dimensional objects. • The length of a curve can be found by combining the distance formula and integration. • There are physical applications of integration in science and statistics.	• What is the relationship between velocity and displacement? • What is the difference between displacement and total distance? • How are velocity, displacement and distance expanded to more general situations? • How do you find the area between two curves? • What are the methods for finding the volume of a solid? • How are they related to finding the area between two curves? • How to you combine the distance formula with integration to find the length of a curve? • Can you find the length of a curve using parametric equations? • What are the physical applications of integration in science and statistics?

Assessment Evidence

Performance Tasks:	**Other Evidence:**
Checkpoints	MyMathLab Remediation
Homework	Skill Refresher
MyMathLab Assignments	Review and Refresh exercises
AP Examination Preparation	Vocabulary concept check

AP Free Response Preparation Lesson Quizzes Mid-Chapter Quiz Exit / Admit Tickets	Study Strategies Calculus Workbook Help

Benchmarks:
Chapter 7 Assessment
AP Chapter 7 Free Response Assessment

Learning Plan

Learning Activities:
*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
Guided notes
Ed Puzzle Notes – optional
Class discussions
Collaborative group work/discussions
Checkpoint/Independent practice
Exit tickets
Homework (online MyMathLab)

Unit Content:

7.1 Accumulation and Net Change
- Apply the definite integral to problems involving motion and use the definite integral to solve problems involving accumulations.
- Model accumulation using Riemann sum approximations.
- Interpret limits of Riemann sums as definite integrals.

7.2 Areas in the Plane
- Apply the definite integral to problems involving area.

7.3 Volumes
- Apply the definite integral to solve problems involving volumes.
- Understand volumes as limits of Reimann sums.
- Solve volume problems with circular, square, cross sections.
- Solve volume of solids of revolution using washers or cylindrical shells.

7.4 Lengths of Curves (BC TOPIC – COVER IF TIME ALLOWS)
- Apply definite integrals to find lengths of smooth curves, lengths of curves with vertical tangents, corners, or cusps.

7.5 Applications from Science and Statistics
- Solve problems involving work, fluid pressure, and probabilities using definite integrals.

Chapter Review

Chapter Test

Resources:

MyMathLab online textbook and practice

QR codes in textbook for access to instructional videos, solutions to exercise and Checkpoint exercises.

MathXL.com for video solutions of selected exercises.

Unit Modifications for Special Population Students	
Advanced Learners	• Invite students to explore different points of view on a topic of study and compare the two. • Assign a leadership role in classroom learning • Determine where student's interests lie and capitalize on their inquisitiveness Expose students to a selection and use of specialized resources
Struggling Learners	• Be flexible with time frames and deadlines • Create planned opportunities for interaction between individuals in the classroom: cooperative and collaborative learning, pair and share with peers • Group students • Intentional scheduling/grouping with student/teacher of alternative background • Provide support as at-risk students move through all levels of knowledge acquisition • Tap prior knowledge

English Language Learners	• Accommodate with completed study guides to assist with preparation on tests • Allow students to give responses in a form (oral or written) that's easier for him/her • Be flexible with time frames, deadlines, or modify assessments • Create planned opportunities for interaction between individuals in the classroom: skits, cooperative and collaborative learning, student generated stories based on personal experience • Establish a framework allowing ELL students to understand and assimilate new ideas and information • Focus on domain specific vocabulary and keywords • Give alternate or paper copies to accommodate electronic assignments. • Have another student share class notes with the ELL student. • Intentional scheduling/grouping with student/teacher of language if possible • Mark texts with a highlighter. • Take more time to complete a task, project, or test. • Use manipulatives, graphic organizer, and real objects when possible • Use visual presentations/verbal materials (ex: word webs and visual organizers).
Special Needs Learners	• Accommodate with completed study guides to assist with preparation on tests. • Allow more time to complete task, project, or test • Allow students to give responses in a form (oral or written) that's easier for him • Be flexible with time frames, deadlines, or modify assessments • Give alternate or paper copies to replace electronic assignments • Have another student share class notes with the special needs learner. • Higher level reasoning and questioning would have less weight than other assignments. • Receive study skill instructions. • Work with fewer items per page or line and/or materials in a larger print.
Learners with a 504	Refer to page four in the Parent and Educator Resource Guide to Section 504 to assist in the development of appropriate plans.

Interdisciplinary Connections

Indicators:

ELA
RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
WHST.11-12.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Computer Science and Design Thinking
8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

Technology Education
9.3.ST.1: Use technology to acquire, manipulate, analyze, and report data.
9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.

Life Literacies & Key Skills
9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
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Integration of 21st Century Skills

Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration and critical thinking.

Unit Title: 8 Sequences, L'Hospital's Rule, and Improper Integrals

Unit Description:
In Chapter 5 students saw how to evaluate definite integrals of continuous functions and bounded functions with finite number of discontinuities on finite closed intervals. These ideas are extended to integrals where one or both limits of integration are infinite, and to integrals whose integrands become unbounded on the interval of integration. Students will see how to use L'Hospital's rule to determine limits on indeterminate forms.

Unit Duration: 3 weeks

Desired Results

Standard(s):
F.BF.A Build a function that models a relationship between two quantities.
F.BF.B Build new functions from existing functions.
A.SSE.A Interpret the structure of expressions.
A.SSE.B Write expressions in equivalent forms to solve problems.
A.APR.A Perform arithmetic operations on polynomials.
A.APR.B Understand the relationship between zeros and factors of polynomials.
A.APR.D Rewrite rational expressions.
F.IF.C Analyze functions using different representations.

Indicators:
F.BF.A.1 Determine an explicit expression, a recursive process, or steps for calculation from a context.
F.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them**.**
A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

- 1a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P
- 2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

A.SSE.B.3-4 3.Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. 4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems.
A.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
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F.IF.C.8-9

- 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Understandings:
Students will understand that…

- There is a rule to determine limits of indeterminate forms. This rule is called, "L'Hospital's rule."
- There are 3 types of indeterminate forms.
- There are techniques we can use to compare growth rates of functions.
- We can use L'Hospital's Rule to compare growth rates.

Essential Questions:

- What is L'Hospital's rule?
- What does it mean to have an indeterminate form?
- How do you identify limits of indeterminate form?
- What is a relative rate of growth?
- How do you compare growth rates of various function types as the independent variable increases without bound?

Assessment Evidence

Performance Tasks:

Checkpoints

Homework

MyMathLab Assignments

AP Examination Preparation

AP Free Response Preparation

Lesson Quizzes

Mid-Chapter Quiz

Exit / Admit Tickets

Other Evidence:

MyMathLab Remediation

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Class discussions
Collaborative group work/discussions
Checkpoint/Independent practice
Exit tickets
Homework (online MyMathLab)

Unit Content:

8.2 L'Hospital's Rule

- Use L'Hospital's Rule to determine limits of indeterminate forms.
- Use L'Hospital's Rule and its stronger form for repeated iterations.

8.3 Relative Rates of Growth

- Compare growth rates of various function types as the independent variable increases without bound.
- Compare growth rates using L'Hospital's Rule.

Chapter Review

Chapter Test

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Unit Modifications for Special Population Students	
Advanced Learners	• Invite students to explore different points of view on a topic of study and compare the two. • Assign a leadership role in classroom learning • Determine where student's interests lie and capitalize on their inquisitiveness Expose students to a selection and use of specialized resources
Struggling Learners	• Be flexible with time frames and deadlines • Create planned opportunities for interaction between individuals in the classroom: cooperative and collaborative learning, pair and share with peers • Group students • Intentional scheduling/grouping with student/teacher of alternative background • Provide support as at-risk students move through all levels of knowledge acquisition
English Language Learners	• Accommodate with completed study guides to assist with preparation on tests • Allow students to give responses in a form (oral or written) that's easier for him/her

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WHST.11-12.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Computer Science and Design Thinking

8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
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8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

Technology Education

9.3.ST.1: Use technology to acquire, manipulate, analyze, and report data.
9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.

Life Literacies & Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

Integration of 21st Century Skills

Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration and critical thinking.