APPC Summer Assignment 1

Calculus

In the late 17th century, Sir Isaac Newton developed a model of Nature that accurately described nearly all the physical phenomena that Renaissance-era natural philosophers and scientists had been exploring since Copernicus first placed the Sun at the center of the Universe. In order to clearly develop his ideas, Newton relied on the tried and true mathematics of Arabia and Greece, i,e. Algebra and Geometry. However, these powerful tools proved to be cumbersome and insufficiently elegant for Newton's new proposed interaction, a.k.a. force. Being the clever Brit that he was, Newton promptly invented a new mathematics. **Yes, he invented Calculus – Yay!**

We found an AP Physics teacher's YouTube channel. His name is Doc Schuster and he does a really good job of explaining the basics.

There is a total of 6 videos and the first 4 are almost 30 minutes each. So, feel free to break this up over a few days.

Here are the details:

- The link: https://www.youtube.com/c/DocSchuster/search?query=ap%20physics%20calculus
- In case that doesn't work, google: *YouTube Channel: Doc Schuster* and search his page for:
    - Titles of Videos: AP Physics Calculus Lessons
    - 6 videos total
        - Intro To Derivatives
        - Why Derivatives are Awesome
        - Intro to Integrals
        - Applications to Physics
        - Definite Integrals
        - Averages and Other Nice Things
- Be ready to take notes. Pretend you are attending a lecture.
- Pause and back it up when necessary. Take your time.

Calculus Problem Sheet #1 Name ____________________

Derivatives

1. $\frac{d\ (3x^2)}{dx} =$

2. $\frac{d\ (5q^2-3q^3+7)}{dq} =$

3. $\frac{d\ (7r+1)}{dr} =$

4. $\frac{d\ (5t^2-7t)}{dt} =$

5. $\frac{d\ (\frac{15z}{z^5})}{dz} =$

6. $\frac{d}{dp}(\frac{7}{p}) =$

7. $\frac{d}{dy}(\frac{3y^5}{y^2}) =$

8. $\frac{d(4)}{dx} =$

9. $\frac{d}{dr}(6r+1)^2 =$

10. $\frac{d\ (\sqrt{t^3})}{dt} =$

Calculus Problem Sheet #2
Indefinite Integrals

1. $\int t^2 \, dt =$

2. $\int (v + 3) dv =$

3. $\int 4t^4 dt =$

4. $3 \int 3 \, dr =$

5. $\int (x^2 + x + 1) dx =$

6. $\int \frac{3}{t^3} \, dt =$

7. $\int \sqrt{m^3} dm =$

8. $\int \frac{dt}{3\sqrt{t}} =$

9. $\int dx =$

10. $\int \frac{y^2}{y^5} dy =$

Calculus Sheet #3
Definite Integrals

1. $\int_0^3 t\,dt =$

2. $\int_{-1}^1 (x^2 - 3x)\,dx =$

3. $\int_2^5 \frac{1}{m^2}\,dm =$

4. $\int_0^3 a\,dt =$
(a is constant)

5. $\int_0^t a\,dt =$

6. $\int_{\sqrt{2}}^{\sqrt{3}} 3t\ \ dt =$

7. $\int_0^3 (7 + 3t)dt =$

8. $\int_{10}^{20} 3m^2\,dm =$

9. $\int_{-2}^2 \frac{1}{y^4}\,dy =$

10. $\int_1^5 \left(3x + x^2 + \frac{x^3}{4}\right) dx =$

APPC Summer Assignment 2 – VECTORS

At some point early on in their study of Physics most students begin to wonder why this course is considered a science class as opposed to another math class...the difference is subtle but important. In science we use mathematics to create accurate, efficient models of physical systems that possess predictive capability. For instance: how high into the air can a person throw a ball? Well, some obvious factors which will affect the outcome of the thrown ball (known as parameters) are things like how fast is it thrown or are there any physical effects opposing the ball's upward motion.

To mathematically describe the motion of an object we employ the techniques of a branch of Physics called "kinematics". We remember that we can pinpoint the location of an object by either using a Cartesian coordinate system (aka "rectangular coordinates") or a Polar coordinate system. We also know how to convert from one system to the other using Pythagoras and trigonometry. The twist for Physics is that we use geometry to represent the actual displacement, velocity, and acceleration of an object experiencing motion by employing vectors.

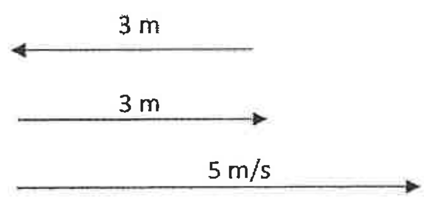
A vector looks like a ray, but the length of the line segment drawn to scale represents the magnitude and the arrowhead indicates direction. This is true for any physical quantity that possesses both magnitude and direction...such as displacement, velocity, or acceleration.

Here's an example *(in which 1 cm = 1 m)*

Displacement vector, 3 m to the left

Displacement vector, 3 m to the right

Velocity vector, 5 m/s to the right



(And remember, a vector is meaningless without UNITS.)

Now...if you experience a series of changes in position (displacements) you can add up the vectors ...2m to the right and 3m to the right and you end up 5m to the right of your original position...however, 3m to the right and 2m to the left and you end up 1 m to the right of your original position.

Makes sense? However, all this "to the right" or "to the left" can be stated more mathematically. In Physics, we use what's called "unit vectors".

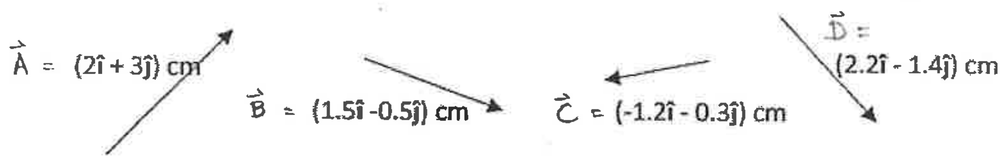
- A unit vector has a magnitude of 1 ...hence the term 'unit' ... and points in either the x, y or z direction.

$\sqrt{x^2+y^2} = \sqrt{14^2+8^2} = 16.1$ m

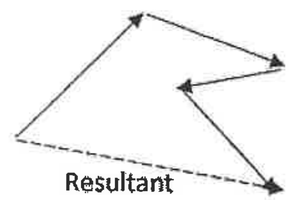
magnitude (size) ↓ units, direction

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{8}{14}\right) = 29.7°$

  - ($14\hat{\imath} + 8\hat{\jmath}$) m ≡ 16.1 m at ~~60.3°~~ 29.7°
  - ($-10\hat{\imath} + 17.3\hat{\jmath}$) m ≡ 20 m at 120° $\sqrt{(-10)^2+(17.3)^2} = 20$ m $\theta = \tan^{-1}\left(\frac{17.3}{10}\right) = 60°$ ∴ 120° From +x axis
- Now what if we wanted to add up several vectors, that looked something like this:



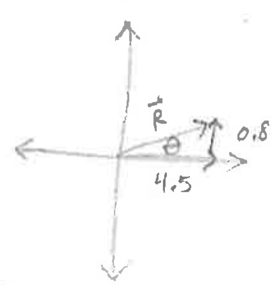
- **Graphically** we add them "head to toe"



- **Mathematically:** add up all the î components, and the ĵ components to get:
  - î direction: 2 + 1.5 -1.2 + 2.2
  - ĵ direction: 3 – 0.5 – 0.3 – 1.4
  - Resultant = $4.5\hat{\imath} + 0.8\hat{\jmath}$
  - Solve for its magnitude using Pythagoras: 4.57 cm
  - Solve for its direction using trigonometry: 10.1°

x	y
2	3
1.5	-0.5
-1.2	-0.3
2.2	-1.4
4.5	+0.8

$\vec{R} = [4.5\hat{\imath} + 0.8\hat{\jmath}]$ cm



$|\vec{R}| = \sqrt{(4.5)^2+(0.8)^2} = 4.57$ cm

$\theta = \tan^{-1}\left(\frac{0.8}{4.5}\right) = 10.1°$ From +x axis

The assignment below allows you to practice vector addition – once you get a few correct you will find that the process is not that difficult. Don't let pride get in the way – you may be a littler rusty or this may be brand new to you – reach out to one another or to me!

Chapter 3 Sections 3.1 - 3.4

Questions: 2, 9, 10, 11, 12

Problems: 1, 2, 3 coordinate system review (if necessary)

To convert between polar and Cartesian coordinates:

$x = r \cos \theta$

$y = r \sin \theta$

Problems: 15, 19, 23, 25. 29, 31, 34, 57

APPC Summer Assignment Part 3

1-D Kinematics

Welcome to the final portion of the summer assignment...

You now have the fundamentals of differential and integral Calculus under your belts, you can manipulate vectors, and you realize that Physics is all about creating mathematical models that allow us to describe physical systems. That's a pretty good start ... We can actually do some Physics now!

Let's get started...

Motion is a relative physical quantity, so the degree of motion experienced by a body is dependent upon the reference frame in which measurements are being made. An inertial frame of reference is one in which Newton's laws of Motion hold true...a non-accelerating reference frame, i.e. a place where the velocity is constant or zero.

Where is the velocity zero? Well ... Never forget that we live on a rotating Earth that is revolving around a Star that is moving about the black hole at the center of a galaxy that is revolving about the center of the local galactic supercluster and all of it is expanding at an incredible rate of speed. . .we don't worry about all these relative motions when we make measurements in the laboratory because all that is in the room around you is moving at the same rate so, relative to you, it is not moving at all!

To hopefully make this point clearer, think about the following scenario. Look at the page before you...relative to you the paper is at rest, v = 0 m/s, but relative to a frame of reference that includes the entire planet that paper is moving very fast...

Imagine this:

- You can sip hot coffee while driving in a car because the cup is at rest relative to your mouth
- You can even share it with a passenger in the car.
- But just try to share that cup of joe with a person standing on the side of the road as you zoom by!
- So, is the cup of coffee at rest or is it in motion?
- The only correct answer is "the state of motion of the cup is dependent upon the frame of reference in which we perform a measurement"
    - Relative to you, the driver, the cup of coffee is at rest, relative to a bystander on the side of the road the cup of coffee is moving....
    - Motion is a **relative physical quantity**
        - just like the rate at which time passes or spatial measurements like distance
        - *but that's all about Einstein's theory of relativity which we won't get into now*

Let me pause at this point to encourage you to continue putting forth effort to understand these ideas...**the only way to learn Physics is by doing it,** immersing yourself in the concepts being presented and realizing that this is more than just fun. We are beginning to understand the way Nature behaves...this is not make-believe or some reasoned philosophical idea...the knowledge base and the skills you develop in Physics are supported by the actual behavior of physical systems, we are learning how the world works!

A review (for most of you) / Good reading (for all of you):

Position - location with respect to an established origin, position is described using coordinates systems, for our purposes Cartesian or polar coordinates

Displacement - change in position in a specified direction displacement is a vector quantity, i.e. to describe a displacement requires a magnitude and a direction

Velocity - time rate change in position, i.e. how fast you are moving in a specified direction velocity is also a vector quantity

Acceleration - time rate change in velocity (also a vector quantity) there are three ways to accelerate: going faster, going slower, and changing direction (since velocity is a vector quantity changing direction, even if your speed is constant, is an example of accelerated motion)

Newton's Laws of Motion:

In the absence of an agent to affect change, the motion of a body will remain constant. In other words:

- an object at rest stays at rest an object in motion stays in motion.

This characteristic of moving bodies was first proposed by Galileo during the 16th Century...it really is quite an extraordinary insight into the behavior of matter. . . because

- if you shove a chair it doesn't keep moving unless you keep pushing it...this is because the floor interacts in such a way with the chair that it impedes the motion of the chair,
- you may recognize this type of interaction as friction...
- in fact, the shove or the push is an interaction, as is gravity or the attraction between oppositely charged particles like protons and electrons...
- these interactions are called forces.
- Whenever two bodies interact upon one another a force of some identifiable type is involved. If multiple forces act upon an object we can perform vector addition (since force is a vector quantity) and determine the resultant force or net force acting on the object.
- If the resultant force is zero, whatever is happening keeps happening (stays at rest or stays in motion) but.....
- if the resultant force is nonzero...aaahhh!, then the body experiencing the unbalanced forces will accelerate! Note the cause and effect relationship....non-zero force causes acceleration.

The above paragraph contains all three of Sir Isaac Newton's Laws of Motion...during the 17th Century, Newton first proposed the concept of force...that bodies interacted and when the net interaction upon a particular body was non-zero, the state of motion of the body would change. Force is a Newtonian idea that revolutionized our way of interpreting the natural world...not until Albert Einstein's relativity would such a radical paradigm shift again occur.

The description of a body's motion without reference to force is a perfectly legitimate exercise since motion does not require force; (**change** in motion (a.k.a. acceleration!) requires force)...kinematics is the description of motion...when forces are taken into account we call it dynamics.

KINEMATICS IN ONE-DIMENSION

The mathematical equations that describe motion in one dimension are quite simple. In fact, I am going to provide you with three equations that are very useful when an object is experiencing motion involving uniform or constant acceleration...when you arrive back to school I would like to begin the course with the development of these equations so that you can appreciate their limited usefulness. . . We can then jump right into Calculus and applications in which the rate of accelerations is time variant. But for now.... When analyzing one dimensional, (i.e. linear motion), that occurs at a constant rate of acceleration, (very common motion when including gravitational freefall) there are five kinematics variables:

- v_o - initial velocity of the body
- v - the velocity of the body after the passage of some non-zero time, t
- a - the acceleration of the body, (for now assume constant acceleration in all applications)
- t - the time during which the motion of the body is being analyzed
- x - the displacement experienced by the body...the final position minus the initial position

The following kinematics equations are only applicable when the acceleration is constant.

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Did I happen to mention that the following kinematics equations are only applicable when the acceleration is constant?

$$v = v_o + at$$

$$v^2 = {v_0}^2 + 2a\,\Delta x$$

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

The next page is an example problem ...

Example #1: Jack can run down a hill with a constant acceleration of 2.0 m/s². If he starts from rest and runs for 10.2 seconds, how far has he traveled at the end of the 10.2 seconds?

Okay first things first...we really don't care how far Jack has run. In fact, Jack doesn't even exist. He is just a character from an old nursery rhyme, (Jill, the hill, and the breaking of crowns). So, what we are looking to do here is to **develop quality problem solving techniques**...

- read the problem statement carefully,
- identify the relevant parameters,
- select the appropriate mathematical model,
- perform mathematical adjustments if necessary, and
- provide a response with appropriate units

Identify relevant parameters: $a = 2.0\ m/s^2$; $v_0 = 0\ m$; $t = 10.2$ sec; $\Delta x = ?$

Select appropriate mathematical model:

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

Perform mathematical adjustments: all information is given with appropriate units

Provide a response: $\Delta x = (1/2)(2)(10.2)^2 = \boxed{104\ m}$

(if you were handwriting this, you would put a box around your answer.)

List + identify

$V_0 = 0$ (From Rest)

$\Delta X = ?$

$a = 2.0\ m/s^2$

$t = 10.2\ s$

Write Appropriate equation

$\Delta X = V_0 t + \frac{1}{2}at^2$

$\Delta X = \frac{1}{2}at^2$

$= \frac{1}{2}\left(2.0\ \frac{m}{s^2}\right)\left(10.2\ s\right)^2$

$\boxed{\Delta X = 104\ m}$

Example #2: A book is dropped off a table. (Assume the effects of the air are negligible -an appropriate assumption for this system). The book will fall 80 cm from the table to the floor. How long does it take for the book to hit the floor and with what velocity does the book hit the floor? (Please note: Galileo was the first to understand and demonstrate that all objects experiencing gravitational freefall accelerate at the same rate...9.8 m/s² directed towards the ground so we will assign a negative sign to the acceleration due to gravity, since it is customary to make "up" the positive direction.)

↑ +
↓ −

Identify relevant parameters: $a = g = -9.8\ m/s^2$; $v_0 = 0$ m; $\Delta x = -80$ cm; t = ?

List
All are vectors and HAVE a direction
$V_0 = 0$
V
$\Delta y = -80\ cm = -0.80\ m$
$a = g = -9.8\ m/s^2$ (acceleration due to gravity)
$t = ?$

Select appropriate mathematical model:

$$\Delta y = v_0 t + \frac{1}{2} g t^2$$

Perform mathematical adjustments:

convert 80 cm to meters: 80 cm = 0.80 m

Rewrite equation to solve for t: $t = \sqrt{\frac{2\,\Delta y}{a}}$

Same equation as before but specific to Freefall
$\Delta x = V_0 t + \frac{1}{2} a t^2$ becomes
$\Delta y = V_0 t + \frac{1}{2} g t^2$ ($V_0 = 0$)
Solve before plugging in
$t = \sqrt{\frac{2\,\Delta y}{g}}$
$t = \sqrt{\frac{2(-0.8\ m)}{-9.8\ m/s^2}}$
$t = 0.4s$

Provide a response:

t = 0.4 s

For second part: v = ?

Select appropriate mathematical model:

$$v^2 = v_0^{\ 2} + 2g\,\Delta y$$

(we could have used $v = v_0 + at$, since we just solved for t. But, when possible, use the information given in the problem, in case you solved for t incorrectly.)

$V_0 = 0$
$V = ?$
$\Delta y = -0.8m$
$g = -9.8\ m/s^2$
t

$V^2 = V_0^2 + 2g\Delta y$ ($V_0 = 0$)
$V = \sqrt{2g\Delta y}$
$V = \sqrt{2(-9.8\ m/s^2)(-0.8\ m)}$
$V = \pm 3.96\ m/s$ * choose the Neg since it is moving downward.
$V = -3.96\ m/s$

No adjustments needed.

Provide a response:

v = -3.96 m/s

Here are the problems involving one dimensional kinematics: (For many of you, this is a review)

Chapter 2: Sections 2.1 through 2.8 READING

Questions: 3, 7, 10, 14

Quick Quiz: 2.2,2.3,2.5,2.7 (quick quizzes are located throughout the chapter and are intended to guide the reader...the quick quiz answers are at the end of the chapter after the problems)

Problems: 23, 27, 29, 38, 41, 51