| Course Title: | Pre-Calculus |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Grade Level(s): | 11-12 |  |  |  |  |
| Duration: | Full Year: | x | Semester: | Marking Period: |  |
| Course Description: | This course builds upon the skills learned in Algebra 1 and Algebra 2. This course also extends into topics that expand Geometry. Linear and quadratic functions from Algebra 1 and Algebra 2 are expanded into higher level topics to connect to real-life problem solving. Exponential and logarithmic functions from Algebra 2 are developed and extended into real-life problem solving. Trigonometric functions such as radian measure, unit circle, graphs of trigonometric functions are expanded to inverse trigonometric functions and composite functions. Applications and models of trigonometric functions are explored. Trigonometric identities are introduced and verified to lead to solving trigonometric functions as well as the use of sum and difference formulas as well as multiple-angle and product to sum formulas. The law of sines and cosines are used to problem solve real-life problems. Vectors are introduced and explored as well as the dot products and the complex plane. Systems of equations are expanded to multivariable systems. Systems of inequalities and linear programming are explored. Matrices are introduced and explored to be used as a tool to solve systems of equations. |  |  |  |  |
| Grading Procedures: | Each semester will be a composite of quiz scores, test scores, homework, and participation reflecting a student's mastery of the areas outlined above. The student can pass the course with an overall average of $70 \%$. The individual teacher will explain the grading system to the student. |  |  |  |  |
| Primary Resources: | NJDOE HS Algebra Model Curriculum <br> NJ Student Learning Standards Mathematics (NJSLS-M) <br> PRECALCULUS with CalcChat and CalcView 11 ${ }^{\text {th }}$ edition, Ron Larson; Cengage Learning, LLC. |  |  |  |  |

## Washington Township Principles for Effective Teaching and Learning

- Implementing a standards-based curriculum
- Facilitating a learner-centered environment
- Using academic target language and providing comprehensible instruction
- Adapting and using age-appropriate authentic materials
- Providing performance-based assessment experiences
- Infusing 21st century skills for College and Career Readiness in a global society

Patricia Pinder

Under the Direction of: Dr. Carole English

## Written:

7/30/2022

## Revised:

BOE Approval: $\qquad$

## Unit 1: Functions and Their Graphs

## Unit Description:

This unit builds upon Algebra 1 and Algebra 2 skills requiring the coordinate plane, linear models, families of functions, operations on functions and inverse functions. Students will extend previous knowledge on families of functions to step functions and piecewise functions. The Difference Quotient is introduced as a way to evaluate functions. Average rate of change is used to extend rate of change to non-linear functions. Students will build upon function operations to compositions and use compositions to prove if two functions are inverse functions. Real-life problems will be graphed on coordinate planes and Algebraic Models will be calculated to represent the data.

## Unit Duration: 3 weeks

## Desired Results

## Standard(s):

A-REI.D Represent and solve equation and inequalities graphically
F-IF.A Understand the concept of a function and use function notation.
F-IF.B Interpret functions that arise in applications of the context.
F-IF.C Analyze functions using different representations.
F-BF. A Build a function that models a relationship between two quantities
F-BF.B Build new functions from existing functions.
S-ID.C Interpret linear models.
G-GPE.A Translate between the geometric descriptions and the equation of a conic section.

## Indicators:

A-REI.D. 10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
F-IF.A. 1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y=f(x)$.
F-IF.A. 2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F-IF.B. 6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F-IF.C.7a, 7b Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ${ }^{\star}$
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-BF.A.1a, 1b, 1c Write a function that describes a relationship between two quantities. ${ }^{\star}$
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
c. Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
F-BF.B.4a, 4b, 4c Find inverse functions.
a. Solve an equation of the form $f(x)=c$ for a simple function $f$ that has an inverse and write an
expression for the inverse. For example, $f(x)=2 x^{3}$ or $f(x)=(x+1) /(x-1)$ for $x \neq 1$.
b. Verify by composition that one function is the inverse of another.
c. Read values of an inverse function from a graph or a table, given that the function has an inverse.
S-ID.C. 7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data

## Understandings:

## Students will understand that...

- The coordinate plane can be used to graph points, lines, and functions.
- Distance between two points can be found using the distance formula.
- The midpoint between two points can be found using the midpoint formula.
- Graphs of functions can be made using tables, $x-y$ intercepts, and symmetry
- Equations of circles can be written using a center and a radius.
- Linear functions can be written by using a slope and a point, a slope, and the $y$-intercept or from two points.
- Parallel lines have the same slope.
- Perpendicular lines have slope that are negative reciprocals.
- Functions have domain and range and can be evaluated.
- Piecewise and step functions can be evaluated and graphed.
- Functions can be evaluated using the difference quotient.
- Average rate of change of a function can be found using the average rate of change formula.
- Vertical Line Test can be used to determine if a relation is a function.
- Functions can be even, odd or neither by evaluating for $\mathrm{f}(-\mathrm{x})$.
- Functions belong to a family of functions with a parent function.
- Transformations (translations, dilations, or reflection) can be performed on the graph and equation of a parent function.
- Operations of addition, subtraction, multiplication, and division can be performed on functions.
- Compositions of functions can be performed on functions.
- Inverses of functions can be found algebraically and graphically.
- Inverse functions can be proved by using compositions.
- Inverse function of a graph can be determined by the Horizontal Line Test.
- Mathematical Models can be used to approximate a set of data points algebraically or by using technology.


## Essential Questions:

- What are the essential parts of a coordinate plane and how do you graph linear functions on the coordinate plane?
- What are the key concepts of a functions in a model, in a graph and in a verbal expression?
- How do functions behave over an interval expression?
- What are the families of functions in Algebra and what are their parent functions and how do transformations (translations, dilations, or reflections) affect the graph of a function as well as the rule of the function?
- How do you find combinations of functions?
- What are inverse functions and how can you determine if two functions are inverse functions?
- How can you determine if a graph of a function has an inverse function?
- How do you make mathematical models of linear functions algebraically? With technology?

| Performance Tasks: | Other Evidence: |
| :--- | :--- |
| Checkpoints | WebAssign remediation |
| Homework | Skill Refresher |
| WebAssign assignments | Review and Refresh exercises |
| Lesson Quizzes | Vocabulary concept check |
| Mid-Chapter Quiz | Study Strategies |
| Exit/Admit Tickets | Algebra Help |

## Benchmarks:

Departmental assessment for Chapter 1
Scatterplot research project - Current State of Planet

## Learning Plan

## Learning Activities:

*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
GO Formative Assessments
Guided notes
Ed Puzzle video notes
Class discussions
Collaborative group work/discussions
Checkpoint/Independent practice
Exit tickets
Homework

## Unit Content:

1.1/1.2 Rectangular Coordinates \& Graphs of Equations (this is a review of Algebra 1 \& 2 and should be done in a quick, efficient manner) (2 Days)

- Coordinate plane, plotting points, making scatter plots
- Finding the distance between two points
- Finding the midpoint between two points
- Graphing an equation on a coordinate plane using a table
- Finding $x$ and $y$ intercepts
- Finding symmetry of a function and using symmetry to graph
- Graph circles
1.3 Linear Equations in Two Variables (this is a review of Algebra $1 \& 2$ and should be done in a quick, efficient manner) (1 Day)
- Graph linear functions using slope and y-intercept
- Calculate slope between two points
- Write equations of lines using slope and a point or two points
- Writing equations of lines parallel to a given line
- Writing equations of lines perpendicular to a given line.
1.4 Functions (this is a review of Algebra $1 \& 2$ and should be done in a quick, efficient manner) (2 Days)
- Testing for functions with mapping diagrams, tables and algebraically
- Evaluating functions using function notation.
- Evaluating a piecewise function.
- Finding the domain of a function from an equation.
- Evaluating the difference quotient.
1.5 Analyzing Graphs of Functions (2 Days)
- Finding the domain and range of a function from a graph.
- Determine if a graph is a function by using the vertical line test.
- Finding the zeros of a function.
- Describe function behavior.
- Approximate a relative min/max.
- Find the average rate of change for non-linear functions.
- Determine even/odd functions.
1.6 A Library of Parent Functions (this is a review of Algebra $1 \& 2$ and should be done in a quick, efficient manner) (1 Day)
- Identify and graph linear, quadratic, cubic, square root, and reciprocal functions.
- Identify and graph step and piecewise functions
1.7 Transformations of Functions (this is a review of Algebra $1 \& 2$ and should be done in a quick, efficient manner) (1 Day)
- Transform graphs of parent functions using translations, reflections and nonrigid transformations.
1.8 Combinations of Functions: Composite Functions (1 Day)
- Add, subtract, multiply and divide with functions.
- Find compositions of functions.
- Find the domain of compositions.
1.9 Inverse Functions (2 Day)
- Find the inverse of a function.
- Verify inverse functions.
- Graph a function and its inverse.
1.10 Mathematical Modeling (1 Day)
- Make a scatterplot and calculate the equation of a line of best fit using algebra.
- Use technology to find the regression line of a set of data.

Chapter Review (1 Day)
Test (1 Day)

## Resources:

WebAssign online textbook and practice via LarsonPreCalculus.com
QR codes in textbook for access to instructional videos, solutions to exercise and Checkpoint exercises.
CalcView.com for video solutions of selected exercises.
CalcChat.com for solutions to all odd-numbered exercises and tests.

| Advanced Learners | - Invite students to explore different points of view on a topic of study and compare the two. <br> - Assign a leadership role in classroom learning <br> - Determine where student's interests lie and capitalize on their inquisitiveness. <br> - Expose students to a selection and use of specialized resources |
| :---: | :---: |
| Struggling Learners | - Be flexible with time frames and deadlines <br> - Create planned opportunities for interaction between individuals in the classroom: cooperative and collaborative learning, pair and share with peers <br> - Group students <br> - Intentional scheduling/grouping with student/teacher of alternative background <br> - Provide support as at-risk students move through all levels of knowledge acquisition <br> - Tap prior knowledge |
| English Language Learners | - Accommodate with completed study guides to assist with preparation on tests <br> - Allow students to give responses in a form (oral or written) that's easier for him/her <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Create planned opportunities for interaction between individuals in the classroom: skits, cooperative and collaborative learning, student generated stories based on personal experience <br> - Establish a framework allowing ELL students to understand and assimilate new ideas and information <br> - Focus on domain specific vocabulary and keywords <br> - Give alternate or paper copies to accommodate electronic assignments. <br> - Have another student share class notes with the ELL student. <br> - Intentional scheduling/grouping with student/teacher of language if possible <br> - Mark texts with a highlighter. <br> - Take more time to complete a task, project, or test. <br> - Use manipulatives, graphic organizer, and real objects when possible <br> - Use visual presentations/verbal materials (ex: word webs and visual organizers). |
| Special Needs Learners | - Accommodate with completed study guides to assist with preparation on tests. <br> - Allow more time to complete task, project, or test <br> - Allow students to give responses in a form ( oral or written) that's easier for him <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Give alternate or paper copies to replace electronic assignments <br> - Have another student share class notes with the special needs learner. <br> - Higher level reasoning and questioning would have less weight than other assignments. <br> - Receive study skill instructions. <br> - Work with fewer items per page or line and/or materials in a larger print |
| Learners with a 504 | Refer to page four in the Parent and Educator Resource Guide to Section $\underline{504}$ to assist in the development of appropriate plans. |

## Indicators:

## ELA

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
WHST.11-12.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

## Computer Science and Design Thinking

8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

## Technology Education

9.3.ST.1: Use technology to acquire, manipulate, analyze, and report data.
9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems,

## Life Literacies \& Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

## Integration of $21^{\text {st }}$ Century Skills

## Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration and critical thinking.

## Unit 2: Polynomial and Rational Functions

## Unit Description:

This unit builds upon Algebra 1 and Algebra 2 skills about quadratic functions and polynomial functions. Graphing functions as well as solving polynomial equations will be extended from skills in Algebra 2. Complex numbers will be used to express solutions to polynomial and quadratic equations. Synthetic division with be used to find zeros of polynomials for real and imaginary solutions. Graphs of rational functions will be extended from Algebra 2 to include rational functions with slant-asymptotes. Nonlinear inequalities will be solved and represented in interval notation.
Unit Duration: 4 weeks

## Desired Results

## Standard(s):

N-CN.A Perform arithmetic operations with complex numbers.
N-CN.C Use complex numbers in polynomial identities and equations.
F-IF.C Analyze functions using different representations
A-APR.B Understand the relationship between zeros and factors of polynomials

## Indicators:

N-CN.A. 1 Know there is a complex number $i$ such that $i^{2}=-1$, and every complex number has the form $a+b i$ with $a$ and $b$ real.
N-CN.A. 2 Use the relation $i^{2}=-1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
N-CN.A. 3 Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
N-CN.C. 7 Solve quadratic equations with real coefficients that have complex solutions.
N-CN.C. 8 Extend polynomial identities to the complex numbers. For example, rewrite $x^{2}+4$ as $(x+2 i)(x-2 i)$.
F-IF.C.7a, 7c, 7d Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ${ }^{\star}$
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
a. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
b. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
F-IF.C.8a Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

A-APR.B. 2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x-a$ is $p(a)$, so $p(a)=0$ if and only if $(x-a)$ is a factor of $p(x)$.
A-APR.B. 3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.

## Understandings:

Students will understand that...

- Graphs of quadratics are parabolas
- Equations of quadratic functions can be written in multiple ways to generate graphs of parabolas
- The minimum and maximum value of a quadratic function is applicable to real-life problems.
- Transformations can be used to transform polynomial functions.
- Graphs of polynomial functions can be made by using the leading coefficient test, zeros, and the intermediate value theorem
- Division of polynomials can be done by long division algorithm or by synthetic division (for $\mathrm{x}-\mathrm{k}$ only).
- The Remainder Theorem can be used to evaluate polynomial functions.
- The Factor Theorem can be used to find linear factors of polynomials.
- The imaginary unit i is used to write complex numbers
- Addition, subtraction, and multiplication can be performed on complex numbers
- The complex conjugate can be used to write the quotient of a two complex numbers in standard form.
- The Fundamental Theorem of Algebra can be used to determine the number of zeros a polynomial function has.
- Polynomial functions can have rational real zeros, irrational real zeros, and complex zeros.
- Descartes's Rule of signs can be used find the zeros of polynomials.
- Zeros of polynomials can be used to solve real-life problems.
- Rational functions have special domains.
- Rational functions have vertical and horizontal asymptotes.
- Graphs of rational functions can be used by finding the parts of the rational function.
- Rational asymptotes can have slant-asymptotes.
- Rational functions can be used to model and solve real-life problems.
- Polynomial and rational inequalities can be solved graphically and algebraically.


## Assessment Evidence

## Performance Tasks:

Checkpoints
Homework
WebAssign assignments
Lesson Quizzes
Mid-Chapter Quiz
Exit/Admit Tickets

## Essential Questions:

- What type of symmetry does the graph of $f(x)=a(x-h)^{2}+k$ have and how can you describe the symmetry?
- What are some common characteristics of the graphs of polynomial functions?
- How can you transform the graph of a polynomial function?
- How many turning points can the graph of a polynomial function have?
- How can you use the factors of a polynomial to solve a division problem involving the polynomial?
- How can you determine whether a polynomial equation has a repeated solution?
- How can you determine whether a polynomial equation has imaginary solutions?
- What are some of the characteristics of the graph of a rational function?
- How do you solve polynomial function inequalities and rational function inequalities?


## Performance Tasks:

WebAssign remediation
Skill Refresher
Review and Refresh exercises
Vocabulary concept check
Study Strategies
Algebra Help

## Benchmarks:

Departmental assessment for Chapter 2
Rollercoaster Polynomial Project

## Learning Plan

## Learning Activities:

*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
GO Formative Assessments
Guided notes \& examples
Ed Puzzle video notes
Class discussions
Collaborative group work/discussions
Checkpoints/Independent practice
Exit tickets
Homework

## Unit Content:

2.1 Quadratic Models and Functions (2 Days)

- Sketch graphs of quadratic functions using transformations.
- Sketch graphs of quadratic functions using standard form.
- Finding the vertex and $x$-intercepts of a parabola
- Write a quadratic function given two points.
- Using the maximum and minimum to solve real-life problems.
2.2 Polynomial Functions of Higher Degree (2 Days)
- Sketch the graph of monomial functions
- Using the leading coefficient test to determine end behaviors.
- Find the real zeros of a polynomial function.
- Sketching graphs of polynomial functions using zeros and the end behaviors.
- Use the intermediate value theorem to approximate real zeros.
2.3 Polynomial and Synthetic Division (2 Days)
- Long division of polynomials with and without remainders.
- Synthetic divide polynomials with and without remainders.
- Use the remainder theorem to evaluate polynomials.
- Use repeated synthetic division to factor polynomials.
2.4 Complex Numbers (2 Days)
- Perform operations with complex numbers.
- Find and use conjugates when dividing with complex numbers.
- Simplify radicals using the imaginary unit i.
- Find complex solutions to quadratic equations using the quadratic formula.


### 2.5 Zeros of Polynomials (3 Days)

- Use the Fundamental Theorem of Algebra to find the number of zeros a polynomial function has.
- Use the rational zero test to find a list of possible zeros for a polynomial.
- Solve polynomial equations by making a list of possible zeros, find zeros using synthetic division and/or repeated synthetic division and factoring.
- Write a polynomial function given zeros, real and complex.
- Factor a polynomial given a complex zero.
- Find all the zeros of a polynomial using synthetic division, factoring, and/or the quadratic formula.
- Use Descartes' rule of signs to find the possible number of positive real, negative real, and imaginary solutions to a polynomial.


### 2.6 Rational Functions (3 Days)

- Find the domain of a rational function
- Find vertical asymptotes and horizontal asymptotes of a rational function.
- Sketch rational functions using asymptotes, intercepts and point between the zeros of a rational function.
- Sketch rational functions with a hole.
- Sketch rational functions with slant asymptotes.
- Solve real-life problems by using rational functions.
2.7 Nonlinear Inequalities (2 Days)
- Solve and graphically represent polynomial inequalities
- Solve and graphically represent rational inequalities.

Chapter Review (1 Day)
Test (1 Day)

## Resources:

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| English Language Learners | - Accommodate with completed study guides to assist with preparation on tests <br> - Allow students to give responses in a form (oral or written) that's easier for him/her <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Create planned opportunities for interaction between individuals in the classroom: skits, cooperative and collaborative learning, student generated stories based on personal experience <br> - Establish a framework allowing ELL students to understand and assimilate new ideas and information <br> - Focus on domain specific vocabulary and keywords <br> - Give alternate or paper copies to accommodate electronic assignments. <br> - Have another student share class notes with the ELL student. <br> - Intentional scheduling/grouping with student/teacher of language if possible <br> - Mark texts with a highlighter. <br> - Take more time to complete a task, project, or test. <br> - Use manipulatives, graphic organizer, and real objects when possible <br> - Use visual presentations/verbal materials (ex: word webs and visual organizers). |
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| Learners with a 504 | Refer to page four in the Parent and Educator Resource Guide to Section $\underline{504}$ to assist in the development of appropriate plans. |

## Indicators:

## ELA

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
WHST.11-12.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

## Computer Science and Design Thinking

8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.
8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

## Technology Education

9.3.ST.1: Use technology to acquire, manipulate, analyze, and report data.
9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems

## Life Literacies \& Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

## Integration of $21^{\text {st }}$ Century Skills

## Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration, critical thinking, and creativity.

## Unit 3: Exponential and Logarithmic Functions

## Unit Description:

This unit builds upon Algebra 1 and Algebra 2 skills about exponential and basic logarithmic functions. Students will graph exponential and logarithmic functions. Students will analyze the effects of transformations on graphs of exponential and logarithmic functions. Domain and range of exponential and logarithmic functions will be compared. Properties of logarithmic expressions will be used to help simplify logarithmic expressions. Exponential and logarithmic equations will be used to solve real-life problems.
Unit Duration: 3 weeks

## Desired Results

Standard(s):
A-REI.A Understand solving equations as a process of reasoning and explain the reasoning
A-SSE.A Interpret the structure of expressions
A-SSE.B Write expressions in equivalent forms to solve problems
F-IF.A Understand the concept of a function and use function notation
F-IF.C Analyze functions using different representations
F-LE.A Construct and compare linear and exponential models and solve problems
F-LE.B Interpret expressions for functions in terms of the situation they model

## Indicators:

A-REI.A. 1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
A-SSE.A.1a, 1b Interpret expressions that represent a quantity in terms of its context. ${ }^{1}$
a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^{n}$ as the product of $P$ and a factor not depending on $P$
A-SSE.B.3c Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *
c. Use the properties of exponents to transform expressions for exponential functions. For example, the expression $1.15^{\mathrm{t}}$ can be rewritten as $\left(1.15^{1 / 12}\right)^{12 t}$ $\approx 1.012^{12 t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is $15 \%$.

F-IF.A. 1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y=f(x)$.
F-IF.C.7e Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ${ }^{\star}$
c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
F-IF.C.8b Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y=$ $(1.02)^{t}, y=(0.97)^{t}, y=(1.01)^{12 t}, y=(1.2)^{t / 10}$, and classify them as representing exponential growth or decay.

F-LE.A.1c Distinguish between situations that can be modeled with linear functions and with exponential functions.
a. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
F-LE.A. 4 Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $a b^{c t}=d$ where $a, c$, and $d$ are numbers and the base $b$ is 2,10 , or $e$; evaluate the logarithm using technology.
F-LE.B. 5 Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

## Understandings:

Students will understand that...

- Exponential functions have a horizontal asymptote
- The natural base e is a value that occurs when exponential functions are expanded over time.
- Logarithmic functions are the inverse function of the exponential function.
- Transformations affect the graphs of exponential and logarithmic functions.
- Logarithmic functions have properties related to the exponent properties to expand and condense.
- Exponential and logarithmic equations can be solved by using one-to one properties, inverse properties, and properties of logarithms.
- Real-life problems can be solved using exponential and logarithmic models.


## Assessment Evidence

| Performance Tasks: | Other Evidence: |
| :--- | :--- |
| Checkpoints | WebAssign remediation |
| Homework | Skill Refresher |
| WebAssign assignments | Review and Refresh exercises |
| Lesson Quizzes | Vocabulary concept check |
| Mid-Chapter Quiz | Study Strategies |
| Exit/Admit Tickets | Algebra Help |

## Benchmarks:

Departmental assessment for Chapter 3
Travel Brochure Project

## Learning Activities:

*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
GO Formative Assessments
Guided notes \& examples
Ed Puzzle video notes
Class discussions
Collaborative group work/discussions
Checkpoints/Independent practice
Exit tickets
Homework

## Unit Content:

### 3.1 Exponential Functions and Their Graphs (2-3 days)

- Evaluating exponential functions
- Graphs of exponential functions
- One-to-One property of exponential functions
- Transformation of graphs of exponential functions
- Natural base e and graphs of natural base e
- One-to-one property of natural base e
- Real-life problems involving exponential functions, including compound interest


### 3.2 Logarithmic Functions (2-3 Days)

- Evaluating logarithmic expressions
- Inverse and one-to-one properties of logarithms
- Graphs of logarithms from exponential graphs and with transformations
- Natural logarithmic functions
- Inverse and one-to-one properties of natural logarithms
- Finding domains of logarithmic functions
- Applications of logarithmic functions


### 3.3 Properties of Logarithms (2 Days)

- Change-of-base formula
- Properties of logarithms
- Expanding logarithms
- Condensing logarithms


### 3.4 Exponential and Logarithmic Functions (2 Days)

- Solving exponential equations using same base and using logarithms
- Solving exponential equations of the quadratic type
- Solve logarithmic equations using one-to-one property and rewriting into exponential form, check for extraneous solutions
- Applications of exponential and logarithmic equations
3.5 Exponential and Logarithmic Models (1 day)
- Exponential growth and decay applications
- Half-life and carbon dating

Chapter Review (1 Day)
Test (1 Day)
Resources:
WebAssign online textbook and practice via LarsonPreCalculus.com
QR codes in textbook for access to instructional videos, solutions to exercise and Checkpoint exercises.
CalcView.com for video solutions of selected exercises.
CalcChat.com for solutions to all odd-numbered exercises and tests.

| Advanced Learners | - Invite students to explore different points of view on a topic of study and compare the two. <br> - Assign a leadership role in classroom learning <br> - Determine where student's interests lie and capitalize on their inquisitiveness. <br> - Expose students to a selection and use of specialized resources |
| :---: | :---: |
| Struggling Learners | - Be flexible with time frames and deadlines <br> - Create planned opportunities for interaction between individuals in the classroom: cooperative and collaborative learning, pair and share with peers <br> - Group students <br> - Intentional scheduling/grouping with student/teacher of alternative background <br> - Provide support as at-risk students move through all levels of knowledge acquisition <br> - Tap prior knowledge |
| English Language Learners | - Accommodate with completed study guides to assist with preparation on tests <br> - Allow students to give responses in a form (oral or written) that's easier for him/her <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Create planned opportunities for interaction between individuals in the classroom: skits, cooperative and collaborative learning, student generated stories based on personal experience <br> - Establish a framework allowing ELL students to understand and assimilate new ideas and information <br> - Focus on domain specific vocabulary and keywords <br> - Give alternate or paper copies to accommodate electronic assignments. <br> - Have another student share class notes with the ELL student. <br> - Intentional scheduling/grouping with student/teacher of language if possible <br> - Mark texts with a highlighter. <br> - Take more time to complete a task, project, or test. <br> - Use manipulatives, graphic organizer, and real objects when possible <br> - Use visual presentations/verbal materials (ex: word webs and visual organizers). |
| Special Needs Learners | - Accommodate with completed study guides to assist with preparation on tests. <br> - Allow more time to complete task, project, or test <br> - Allow students to give responses in a form ( oral or written) that's easier for him <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Give alternate or paper copies to replace electronic assignments <br> - Have another student share class notes with the special needs learner. <br> - Higher level reasoning and questioning would have less weight than other assignments. <br> - Receive study skill instructions. <br> - Work with fewer items per page or line and/or materials in a larger print |
| Learners with a 504 | Refer to page four in the Parent and Educator Resource Guide to Section $\underline{504}$ to assist in the development of appropriate plans. |

## Indicators:

## ELA

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
WHST.11-12.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

## Computer Science and Design Thinking

8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.
8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

## Technology Education

9.3.ST.1: Use technology to acquire, manipulate, analyze, and report data.
9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems

## Financial Literacy

9.1.12.CDM.6: Compute and assess the accumulating effect of interest paid over time when using a variety of sources of credit. (e.g., student loans, credit cards, auto loans, mortgages, etc.).
9.1.12.CDM.8: Compare and compute interest and compound interest and develop an amortization table using business tools.

## Life Literacies \& Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

## Integration of 21 ${ }^{\text {st }}$ Century Skills

## Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration, critical thinking, communication, and creativity.

## Unit 4: Trigonometric Functions

Unit Description: This unit explores angles and radians as well as introduces trigonometric functions in right triangles as well as on the unit circle. Graphs of sine and cosine are reviewed and expanded to translations of the graphs using phase shifts and periods. Graphs of tangent and cotangent are introduced as well as translations of each function. Reciprocal function graphs of secant and cosecant are created by using the graphs of sine and cosine. Inverse trigonometric functions are introduced, and arcsine, arccosine and arctangent are used to evaluate functions as well as evaluate composite functions. Applications involving right triangle trigonometry are used to solve real-life problems involving right triangles.
Unit Duration: 5-6 weeks

## Desired Results

## Standard(s):

F-TF.A Extend the domain of trigonometric functions using the unit circle
F-TF.B Model periodic phenomena with trigonometric functions
G-SRT.C Define trigonometric ratios and solve problems involving right triangles

## Indicators:

F-TF.A. 1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F-TF.A. 2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
F-TF.A. 3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi / 3, \pi / 4$, and $\pi / 6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi x, \pi+x$, and $2 \pi-x$ in terms of their values for $x$, where $x$ is any real number.
F-TF.A. 4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
F-TF.B. 5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
F-IF.B. 7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context.
G-SRT.C. 6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
G-SRT.C. 8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

## Understandings:

Students will understand that...

- Radian measure is another way to measure angles.
- The unit circle can be used to find the six trigonometric functions.
- Properties from the unit circle can be used to find the trigonometric function of any angle.
- Right triangle trigonometry functions can be used to find angles and sides of a triangle.
- Graphs of sine and cosine have similar characteristics.


## Essential Questions:

- How can you find the measure of an angle in radians?
- How can you use the unit circle to define the trigonometric functions of any angle?
- How can you find a trigonometric function of an acute angle $\Theta$ ?
- What are the characteristics of the graph of the sine and cosine function?
- What are the characteristics of the graph of the tangent and cotangent function?
- What are the characteristics of the graph of the secant and cosecant function?
- What are the characteristics of the real-life
- Translations can be performed on graphs of sine and cosine using phase shift, amplitude, and period.
- Tangent and cotangent functions have similar characteristics and can be translated using phase shift, amplitude, and period.
- Reciprocal functions can be graphed by using the sine and cosine functions.
- Real-life problems can be solved with trigonometry.

Assessment Evidence

| Performance Tasks: | Other Evidence: |
| :--- | :--- |
| Checkpoints | WebAssign remediation |
| Homework | Skill Refresher |
| WebAssign assignments | Review and Refresh exercises |
| Lesson Quizzes | Vocabulary concept check |
| Mid-Chapter Quiz | Study Strategies |
| Exit/Admit Tickets | Algebra Help |

## Benchmarks:

Departmental assessment for Chapter 4
Unit Circle Picture Project

Learning Plan

## Learning Activities:

*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
GO Formative Assessments
Guided notes \& examples
Ed Puzzle video notes
Class discussions
Collaborative group work/discussions
Checkpoints/Independent practice
Exit tickets
Homework

## Unit Content:

4.1 Angles and Radian Measure (3-4 Days)

- Describe angles
- Use radian measure, degree measure and convert from degrees to radians and radians to degrees
- Find coterminal angles
- Find complementary and supplementary angles
- Use angles and their measure to model real-life problems
4.2 The Unit Circle (2-3 Days)
- Identify the unit circle and describe its relationship to real numbers
- Evaluate trigonometric functions using the unit circle.
- Use domain and period to evaluate sine and cosine functions.
- Use a calculator to evaluate trigonometric functions
4.3 Right Triangle Trigonometry (3 Days)
- Evaluate trigonometric functions of acute triangles
- Use fundamental trigonometric identities
- Use trigonometric functions to model and solve real-life problems
4.4 Trigonometric Functions of Any Angle (2 days)
- Evaluate trigonometric functions of any angle
- Find reference angles
- Evaluate trigonometric functions of real numbers
4.5 Graphs of Sine and Cosine Functions (2-3 Days)
- Sketch the basic sine and cosine function
- Use amplitude and period to help sketch the graph of sine and cosine functions
- Sketch translations of the sine and cosine functions
- Use sine and cosine functions to model and solve real-life problems.
4.6 Graphs of other Trigonometric Functions (2 Days)
- Sketch the graphs of tangent functions
- Sketch the graph of cotangent functions
- Sketch the graph of secant and cosecant functions
4.7 Inverse Trigonometric Functions (2 Days)
- Evaluate and graph inverse sine functions.
- Evaluate and graph other inverse trigonometric functions.
- Evaluate composite functions involving inverse trigonometric functions.
4.8 Applications and Models of Trigonometric Functions (2 Days)
- Solve real-life problems involving right triangles
- Solve real-life problems involving directional bearings.
- Solve real-life problems involving harmonic motion

Chapter Review (1 Day)
Test (1 Day)

## Resources:

WebAssign online textbook and practice via LarsonPreCalculus.com
QR codes in textbook for access to instructional videos, solutions to exercise and Checkpoint exercises.
CalcView.com for video solutions of selected exercises.
CalcChat.com for solutions to all odd-numbered exercises and tests.

| Advanced Learners | - Invite students to explore different points of view on a topic of study and compare the two. <br> - Assign a leadership role in classroom learning <br> - Determine where student's interests lie and capitalize on their inquisitiveness. <br> - Expose students to a selection and use of specialized resources |
| :---: | :---: |
| Struggling Learners | - Be flexible with time frames and deadlines <br> - Create planned opportunities for interaction between individuals in the classroom: cooperative and collaborative learning, pair and share with peers <br> - Group students <br> - Intentional scheduling/grouping with student/teacher of alternative background <br> - Provide support as at-risk students move through all levels of knowledge acquisition <br> - Tap prior knowledge |
| English Language Learners | - Accommodate with completed study guides to assist with preparation on tests <br> - Allow students to give responses in a form (oral or written) that's easier for him/her <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Create planned opportunities for interaction between individuals in the classroom: skits, cooperative and collaborative learning, student generated stories based on personal experience <br> - Establish a framework allowing ELL students to understand and assimilate new ideas and information <br> - Focus on domain specific vocabulary and keywords <br> - Give alternate or paper copies to accommodate electronic assignments. <br> - Have another student share class notes with the ELL student. <br> - Intentional scheduling/grouping with student/teacher of language if possible <br> - Mark texts with a highlighter. <br> - Take more time to complete a task, project, or test. <br> - Use manipulatives, graphic organizer, and real objects when possible <br> - Use visual presentations/verbal materials (ex: word webs and visual organizers). |
| Special Needs Learners | - Accommodate with completed study guides to assist with preparation on tests. <br> - Allow more time to complete task, project, or test <br> - Allow students to give responses in a form ( oral or written) that's easier for him <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Give alternate or paper copies to replace electronic assignments <br> - Have another student share class notes with the special needs learner. <br> - Higher level reasoning and questioning would have less weight than other assignments. <br> - Receive study skill instructions. <br> - Work with fewer items per page or line and/or materials in a larger print |
| Learners with a 504 | Refer to page four in the Parent and Educator Resource Guide to Section $\underline{504}$ to assist in the development of appropriate plans. |

## Indicators:

## ELA

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
WHST.11-12.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

## Computer Science and Design Thinking

8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.
8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

## Technology Education

9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems

## Life Literacies \& Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).

## Integration of 21 ${ }^{\text {st }}$ Century Skills

## Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration and critical thinking.

## Unit 5: Analytic Trigonometry

Unit Description: This unit builds upon the trigonometric properties from the previous unit. Fundamental trigonometric identities are introduced and used to evaluate trigonometric functions. The process of verifying trigonometric functions is explored as algebraic proofs. Trigonometric equations are solved for simple trigonometric functions and equations involving multiple angles. Sum and difference formulas are introduced to evaluate trigonometric functions. Multiple-Angle and product-to-sum formulas are used to solve multipleangle equations in many forms as well as to write products as sums to solve other trigonometric equations.

## Unit Duration: 3-4 weeks

## Desired Results

## Standard(s):

F-TF.C Prove and apply trigonometric identities

## Indicators:

F-TF.C. 8 Prove the Pythagorean identity $\sin ^{2}(\theta)+\cos ^{2}(\theta)=1$ and use it to find $\sin (\theta), \cos (\theta)$, or $\tan (\theta)$ given $\sin (\theta), \cos (\theta)$, or $\tan (\theta)$ and the quadrant of the angle.
F-FT.C. 9 Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

## Understandings:

Students will understand that...

- Trigonometric identities can be used to simply a trigonometric expression.
- Trigonometric identities can be verified using algebraic proofs.
- Trigonometric functions can be evaluated using the sum or difference of two angles.
- Trigonometric functions can be evaluated using multiple angle formulas.
- Multiple angle formulas can be used to solve trigonometric equations.
- Trigonometric functions can be evaluated using product-to-sum formulas.
- Trigonometric equations can be used to solve using product-to-sum formulas.


## Essential Questions:

- How can you verify a trigonometric identity?
- How do you solve trigonometric equations?
- How can you evaluate trigonometric functions of the sum or difference of two angles?
- How can you evaluate trigonometric functions using multiple angle formula?
- How can you solve trigonometric equations involving multiple angles, double angles, and half angles?
- How can you evaluate trigonometric functions using product-to-sum formulas?

Assessment Evidence

## Performance Tasks:

Checkpoints
Homework
WebAssign assignments
Lesson Quizzes
Mid-Chapter Quiz
Exit/Admit Tickets

## Other Evidence:

WebAssign remediation
Skill Refresher
Review and Refresh exercises
Vocabulary concept check
Study Strategies
Algebra Help

## Benchmarks:

Departmental assessment for Chapter 5
Trigonometric Identities Project

## Learning Plan

Learning Activities:
*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
GO Formative Assessments
Guided notes \& examples
Ed Puzzle video notes
Class discussions
Collaborative group work/discussions
Checkpoints/Independent practice
Exit tickets
Homework

## Unit Content:

### 5.1 Verifying Fundamental Identities (2 Days)

- Using Fundamental Identities
- Factoring, simplifying, adding, and rewriting trigonometric expressions
- Using trigonometric substitution
- Rewriting a logarithmic expression


### 5.2 Verifying Trigonometric Identities (2-3 Days)

- Verifying trigonometric identities
- Converting to sines and cosines


### 5.3 Solving Trigonometric Equations (3 days)

- Collecting like terms
- Extracting square roots
- Factoring
- Equations of quadratic type
- Equations involving multiple angles
- Using inverse functions


### 5.4 Sum and Difference Formulas (2 Days)

- Using sum and difference formulas
- Verifying a cofunction identity
- Deriving reduction formulas
- Multiple-Angle formulas
- Evaluating functions involving double angles
- Deriving a triple-angle formula
- Power-reducing formulas
- Half-angle formulas
- Product-to-sum and sum-to-product formulas
- Applications of multiple-angle and product-to-sum formulas

Chapter Review (1 Day)
Test (1 Day)
Resources:
WebAssign online textbook and practice via LarsonPreCalculus.com
QR codes in textbook for access to instructional videos, solutions to exercise and Checkpoint exercises.
CalcView.com for video solutions of selected exercises.
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| Special Needs Learners | - Accommodate with completed study guides to assist with preparation on tests. <br> - Allow more time to complete task, project, or test <br> - Allow students to give responses in a form ( oral or written) that's easier for him <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Give alternate or paper copies to replace electronic assignments <br> - Have another student share class notes with the special needs learner. <br> - Higher level reasoning and questioning would have less weight than other assignments. <br> - Receive study skill instructions. <br> - Work with fewer items per page or line and/or materials in a larger print |
| Learners with a 504 | Refer to page four in the Parent and Educator Resource Guide to Section $\underline{504}$ to assist in the development of appropriate plans. |

## Indicators:

## ELA

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
WHST.11-12.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

## Computer Science and Design Thinking

8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

## Technology Education

9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems

## Life Literacies \& Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).

## Integration of $21{ }^{\text {st }}$ Century Skills

## Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration and critical thinking.

| Unit 6: Additional Topics in Trigonometry |
| :--- |
| Unit Description: This unit covers additional topics in trigonometry such as the Law of Sine and Cosines with <br> applications of both. The area of an oblique triangle and Heron's area of a triangle are introduced to find the <br> area of any triangle. Applications of the Law of Sines and Cosines are explored. Vectors are introduced and <br> the characteristics are explored. Vector operations and unit vectors are introduced to solve applications of <br> vectors. The Dot Product of two vectors are introduced to find the angle between two vectors as well as find <br> an orthogonal vector. The complex plane is used to find operations with complex numbers, the distance <br> between two complex numbers, and the midpoint in the complex plane. Trigonometric form of a complex <br> number is introduced and explored over operations, powers of complex numbers and DeMoivre's Theorem, <br> and roots of a complex number. <br> Unit Duration: 3-4 weeks |

## Desired Results

## Standard(s):

G-STR.D Apply trigonometry to general triangles
N-VM.A Represent and model with vector quantities
N-VM.B Perform operations on vectors.
N-CN.B Represent complex numbers and their operations on the complex plane.

## Indicators:

G-SRT.D. 9 Derive the formula $A=1 / 2 a b \sin (C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G-SRT.D. 10 Prove the Laws of Sines and Cosines and use them to solve problems.
G-SRT.D. 11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
N-VM.A. 1 Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v},|\boldsymbol{v}|,\|\boldsymbol{v}\|, \boldsymbol{v}$ ).
N.VM.A. 2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
N.VM.A. 3 Solve problems involving velocity and other quantities that can be represented by vectors.
N-VM-B.4a, 4b Add and subtract vectors.
a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
N-VM.B.5a, 5b Multiply a vector by a scalar.
a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v x, v y)=(c v x, c v y)$.
b. Compute the magnitude of a scalar multiple $c v$ using $\|c v\|=|c| v$. Compute the direction of $c \boldsymbol{v}$ knowing that when $|c| \boldsymbol{v} \neq 0$, the direction of $c \boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c>0$ ) or against $\boldsymbol{v}$ (for $c<0$ ).
N-CN.B. 4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number.

N-CN.B. 5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 \pm 3 i)^{3}=8$ because ( $-1 \pm 3 \mathrm{i}$ ) has modulus 2 and argument $120^{\circ}$.

## Understandings:

Students will understand that...

- The Law of Sines can be used to solve triangles.
- The Law of Sines can be used to find the area of oblique triangles.
- The Law of Cosines can be used to solve triangles.
- Heron's area formula can be used to find the area of oblique triangles.
- A vector can be used represent magnitude and direction.
- Operations can be performed on vectors.
- Dot Products can be used to find the angle between vectors and to determine if vectors are orthogonal.
- The complex plane can be used to write complex numbers, perform operations on complex numbers and find trigonometric form of complex numbers.
- Trigonometry of complex numbers can be used to find powers and roots of complex numbers.


## Essential Questions:

- How do you use the Law of Sines to solve triangles?
- How do you use the Law of Cosines to solve triangles?
- What is a vector?
- How do you use vectors to perform operations such as addition and subtraction?
- How do you find the Dot Product of vectors?
- How do you use the Dot Product to find the angle between the vectors?
- How do you use the Dot Product to determine if vectors are orthogonal?
- How do you plot complex numbers on the complex plane?
- How do you use the complex plane to perform operations on complex numbers?
- How do you find the trigonometric form of a complex number?
- How do you perform operations on complex numbers in the complex plane?
- How do you find a power of a complex number?
- How do you find the roots of complex numbers?

Assessment Evidence

| Performance Tasks: | Other Evidence: |
| :--- | :--- |
| Checkpoints | WebAssign remediation |
| Homework | Skill Refresher |
| WebAssign assignments | Review and Refresh exercises |
| Lesson Quizzes | Vocabulary concept check |
| Mid-Chapter Quiz | Study Strategies |
| Exit/Admit Tickets | Algebra Help |

## Benchmarks:

Departmental assessment for Chapter 6
Law of Sines and Cosines Diorama project

## Learning Activities:

*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
GO Formative Assessments
Guided notes \& examples
Ed Puzzle video notes
Class discussions
Collaborative group work/discussions
Checkpoints/Independent practice
Exit tickets
Homework

## Unit Content:

### 6.1 Law of Sines (2 Days)

- Use the law of sines to solve oblique triangles (AAS or ASA)
- Use the law of sine to solve oblique triangles (SSA)
- Find the area of oblique triangles
- Use the law of sine to model and solve real-life problems


### 6.2 Law of Cosines (2 Days)

- Use the law of cosines to solve oblique triangles (SSS or SAS)
- Use the law of cosines to model and solve real-life problems
- Use Heron's area formula to find area of triangles


### 6.3 Vectors in the Plane (2-3 Days)

- Represent vectors as directed line segments
- Write component forms of vectors
- Perform basic vector operations and represent graphically.
- Write vectors as linear combinations of unit vectors
- Find direction angles of vectors
- Use vectors to model and solve real-life problems.


### 6.4 Vectors and Dot Products (2 Days)

- Find the dot product of two vectors and use the properties of the dot product.
- Find the angle between two vectors and determine whether two vectors are orthogonal.
- Write a vector as the sum of two vector components
- Use vectors to determine the work done by a force.


### 6.5 The Complex Plane (2 Days)

- Plot complex numbers in the complex plane and find absolute values of complex numbers
- Perform operations with complex numbers in the complex plane
- Use the distance and midpoint formulas in the complex plane


### 6.6 Form of a Complex Number (2 Days)

- Write trigonometric forms of complex numbers.
- Multiply and divide complex numbers written in trigonometric form.
- Use DeMoivre's Theorem to find powers of complex numbers.
- Find $n^{\text {th }}$ roots of complex numbers.

Chapter Review (1 Day)
Test (1 Day)

## Resources:

WebAssign online textbook and practice via LarsonPreCalculus.com
QR codes in textbook for access to instructional videos, solutions to exercise and Checkpoint exercises.
CalcView.com for video solutions of selected exercises.
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## Unit Modifications for Special Population Students

| Advanced Learners | - Invite students to explore different points of view on a topic of study and compare the two. <br> - Assign a leadership role in classroom learning <br> - Determine where student's interests lie and capitalize on their inquisitiveness. <br> - Expose students to a selection and use of specialized resources |
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| English Language Learners | - Accommodate with completed study guides to assist with preparation on tests <br> - Allow students to give responses in a form (oral or written) that's easier for him/her <br> - Be flexible with time frames, deadlines, or modify assessments <br> - Create planned opportunities for interaction between individuals in the classroom: skits, cooperative and collaborative learning, student generated stories based on personal experience <br> Establish a framework allowing ELL students to understand and assimilate new ideas and information <br> - Focus on domain specific vocabulary and keywords <br> - Give alternate or paper copies to accommodate electronic assignments. <br> - Have another student share class notes with the ELL student. <br> - Intentional scheduling/grouping with student/teacher of language if possible <br> - Mark texts with a highlighter. <br> - Take more time to complete a task, project, or test. <br> - Use manipulatives, graphic organizer, and real objects when possible <br> - Use visual presentations/verbal materials (ex: word webs and visual organizers). |
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## ELA

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
WHST.11-12.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

## Computer Science and Design Thinking

8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

## Technology Education

9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems

## Life Literacies \& Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).

## Integration of $21{ }^{\text {st }}$ Century Skills

## Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration and critical thinking.

| Unit 7: Systems of Equations and Inequalities |
| :--- |
| Unit Description: This unit extends the methods of solving systems of equations in two-variables to non- |
| linear systems. Substitution and elimination are explored and expanded to real-world models. Multivariable |
| linear systems are solved algebraically. Gaussian elimination and row-echelon form are introduced and used |
| to solve systems of linear equations. Special solutions of systems of equations are represented algebraically |
| and graphically. Applications of systems using multivariable as well as linear and non-square systems will be |
| solved. Partial fractions are introduced. Decomposition of fractions to partial fractions is explored into distinct |
| linear factors, repeated linear factors, distinct linear and quadratic factors, repeated quadratic factors, and |
| repeated linear and quadratic factors. Systems of inequalities are represented graphically and solved |
| graphically. Special solutions of systems of inequalities are explored including bounded and unbounded |
| solution sets. Linear programming is introduced and explored to solve real-life problems. |

Unit Duration: 3-4 weeks

## Desired Results

## Standard(s):

A-REI.C Solve systems of equations
A-REI.D Represent and solve equations and inequalities graphically

## Indicators:

A-REI.C. 5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
A-REI.C. 6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
A-REI.C. 7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y=-3 x$ and the circle $x^{2}+y^{2}=3$.
A-REI.D. 10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A-REI.D. 12 Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding halfplanes.

## Understandings:

Students will understand that...

- Systems of equations can be solved by using substitution.
- Systems of equations can be solved by using elimination.
- Systems of equations have special solutions.
- Systems of equations with multivariables can be solved using substitution.
- Systems of equations with multivariables can be solved using Gaussian elimination.
- Non-square linear systems solutions do not have one exact solution.
- Partial fractions can be used to decompose fractional expressions.
- Systems of inequalities can be solved graphically.
- Systems of inequalities can be bounded or unbounded.


## Essential Questions:

- How do you solve a system of equations by using the substitution method?
- How do you solve a system of equations by using elimination?
- How do you represent special solutions to systems of equations graphically and algebraically?
- How do you solve multivariable systems of equations?
- How do you use the Gaussian method and row-echelon form to solve systems of equations?
- How can you solve non-square linear systems of equations?
- How can partial fractions be used to decompose fractional expressions?
- How do you solve systems of inequalities?
- How do you use linear programming to find the solution to real-life problems?
- Linear programming can be used to solve systems of inequalities.


## Assessment Evidence

## Performance Tasks:

Checkpoints
Homework
WebAssign assignments
Lesson Quizzes
Mid-Chapter Quiz
Exit/Admit Tickets

## Other Evidence:

WebAssign remediation
Skill Refresher
Review and Refresh exercises
Vocabulary concept check
Study Strategies
Algebra Help

## Benchmarks:

Departmental assessment for Chapter 7
Linear systems of equations project
Linear programming project

## Learning Plan

## Learning Activities:

*Lessons may include some or all of the following activities
Daily Warm Up/Do Nows
Review of Homework
GO Formative Assessments
Guided notes \& examples
Ed Puzzle video notes
Class discussions
Collaborative group work/discussions
Checkpoints/Independent practice
Exit tickets
Homework

## Unit Content:

### 7.1 Linear and Nonlinear of Systems of Equations (1-2 Days)

- Use methods of substitution to solve systems of linear equations in two variables.
- Use methods of substitution to solve systems of nonlinear equations in two variables.
- Use a graphical method to solve systems of equation in two variables
- Use systems of equations to model and solve real-life
problems


### 7.2 Two-Variable Linear Systems (1-2 Days)

- The elimination method
- Recognizing graphs of linear systems
- Special solutions to linear systems
- Solving real-word problems using systems of equations in two variables
7.3 Multivariable Linear Systems (2 Days)
- Row-Echelon form and back-substitution
- Gaussian Elimination
- Non-square systems
- Applications of systems


### 7.4 Partial Fractions (optional)

- Partial fraction decompositions
- Repeated linear factors
- Distinct linear and quadratic factors
- Repeated quadratic factors


### 7.5 Systems of Inequalities (1-2 Days)

- Graphs of inequalities
- Systems of linear inequalities
- System of linear and nonlinear inequalities
- Applications of systems of inequalities


### 7.6 Linear Programming (2 Days)

- Linear Programming - graphically
- Minimizing an objective function
- Maximizing an objective function
- An unbounded region
- Application problems

Chapter Review (1 Day)
Test (1 Day)

## Resources:

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## Unit Modifications for Special Population Students

| Advanced Learners | - Invite students to explore different points of view on a topic of study and compare the two. <br> - Assign a leadership role in classroom learning <br> - Determine where student's interests lie and capitalize on their inquisitiveness. <br> - Expose students to a selection and use of specialized resources |
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| Learners with a 504 | Refer to page four in the Parent and Educator Resource Guide to Section $\underline{504}$ to assist in the development of appropriate plans. |

## Indicators:

## ELA

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text
RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
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## Computer Science and Design Thinking

8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.
8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
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## Technology Education

9.3.ST.1: Use technology to acquire, manipulate, analyze, and report data.
9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems

## Life Literacies \& Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

## Integration of $21^{\text {st }}$ Century Skills

Indicators:
From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration and critical thinking as well as creativity.

| Unit 8: Matrices and Determinants |
| :--- |
| Unit Description: This unit introduces matrices. Elementary row operations are used to find row-equivalent |
| matrices. Gaussian elimination with back-substitution is introduced to solve linear systems. Determinants are |
| introduced and matrices are used to solve systems of equations. Equality of matrices are defined. Simple |
| operations and scalar matrices are performed on matrices. Matrix equations are introduced to solve systems |
| of equations. Matrix multiplication is introduced and expanded to squaring a matrix. Matrices are used to |
| transform vectors. Inverse matrices are defined and are explored to finding an inverse matrix of a $2 \times 2$ matrix. |
| Systems of equations are solved using inverse matrices. Determinants of $2 \times 2$ matrix is explored as well as |
| the minor and cofactors of a square matrix. Determinants of $3 \times 3$ and $4 \times 4$ matrices are also explored. |
| Application of matrices and determinants are used to connect to Cramer's Rule as a method for solving |
| systems of equations. |
| Unit Duration: 3-4 weeks |

## Desired Results

Standard(s):
N-VM.C Perform operations on matrices and use matrices in applications.
A-REI.C Solve systems of equations
Indicators:
N-VM.C. 6 Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
N-VM.C. 7 Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
N-VM.C. 8 Add, subtract, and multiply matrices of appropriate dimensions.
N-VM.C. 9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
N-VM.C. 10 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
N-VM.C. 11 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
N-VM.C. 12 Work with $2 \times 2$ matrices as a transformation of the plane and interpret the absolute value of the determinant in terms of area.
A-REI.C. 8 Represent a system of linear equations as a single matrix equation in a vector variable.
A-REI.C. 9 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension $3 \times 3$ or greater).

## Understandings:

Students will understand that...

- A matrix is an array of numbers that contains rows and columns.
- Coefficient and augmented matrices can be written to represent systems of equations.
- Row-echelon form of a matrix is unique and can be used to help solve systems of equation using the Gaussian back-substitution method for solving systems.
- Matrices have equality and element operations can be performed on matrices such as addition, subtraction, and scalar multiplication.


## Essential Questions:

- What defines a matrix?
- How do you write an augmented matrix?
- How do you perform elementary row operations on a matrix?
- What is row-echelon form and how can you solve a system of equations using Gaussian elimination?
- How do you use matrix equality to solve for variables?
- How do you perform addition and subtraction on matrices?
- How do you use scalar multiplication with matrices?
- How do you solve system of equations using
- Matrix equations can be used to solve systems of equations.
- Multiplication of matrices can only be performed if the number of columns of the first matrix equals the number of rows of the second.
- Vector operations can be done with vectors written in column matrices.
- Two matrices are inverse matrices if the product of the matrices yield the identity matrix.
- Systems of equations can be solved using inverse matrices.
- Every square matrix has a determinant.
- Matrices of dimension $3 \times 3$ or higher has minors and cofactors.
- Cramer's rule is a method for solving matrix equations using determinants.
matrix equations?
- How can you multiply matrices?
- How can you use matrices to perform operations on vectors?
- How can you verify if two matrices are inverse matrices?
- How can you find the inverse of a matrix?
- How can you use inverse matrices to solve a system of linear equations?
- How can you find the determinant of a $2 \times 2$ matrix?
- How can you find the minor and cofactors of square matrices?
- How can you find the determinant of other square matrices?
- How do you use Cramer's Rule to solve systems of linear equations?


## Assessment Evidence

Performance Tasks:
Checkpoints
Homework
WebAssign assignments
Lesson Quizzes
Mid-Chapter Quiz
Exit/Admit Tickets

## Other Evidence:

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Skill Refresher
Review and Refresh exercises
Vocabulary concept check
Study Strategies
Algebra Help

## Benchmarks:

Departmental assessment for Chapter 8
Matrices and systems of equations project

## Learning Plan

## Learning Activities:

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Homework

## Unit Content:

8.1 Matrices and Systems of Equations (2-3 Days)

- Write matrices and determine their dimensions
- Perform elementary row operations on matrices
- Use matrices and Gaussian elimination to solve systems of linear equations
- Use matrices and Gauss-Jordan elimination to solve systems of linear equations


### 8.2 Operations with Matrices (2 Days)

- Determine whether two matrices are equal
- Add and subtract matrices, and multiply matrices by scalars
- Multiply two matrices
- Use matrices to transform vectors
- Use matrix operations to model and solve real-life problems


### 8.3 The Inverse of a Square Matrix (2 Days)

- Verify that two matrices are inverses of each other
- Use Gauss-Jordan elimination to find the inverse of matrices
- Use a formula to find the inverse of $2 \times 2$ matrix
- Use inverse matrices to solve systems of linear equations.


### 8.4 The Determinant of a Square Matrix (1-2 Days)

- Find the determinants of $2 \times 2$ matrices
- Find the minors and cofactors of square matrices
- Find the determinants of square matrices


### 8.5 Applications of Matrices and Determinants (3 Days)

- Use Cramer's Rule to solve systems of linear equations.
- Use determinants to find areas of triangles.
- Use determinants to test for collinear points and find equations of lines passing through two points
- Use $2 \times 2$ matrices to perform transformations in the plane and find areas of parallelograms
- Use matrices to encode and decode messages


## Chapter Review (1 Day)

Test (1 Day)

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8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms
8.2.5.ED.2: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
8.2.5.ED.3: Follow step by step directions to assemble a product or solve a problem, using appropriate tools to accomplish the task.

## Technology Education

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9.3.ST-SM.2: Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems

## Life Literacies \& Key Skills

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

## Integration of 21 ${ }^{\text {st }}$ Century Skills

## Indicators:

From the Partnership for 21st Century Skills (P21), the deeper learning competencies and skills for 21st century learning in this unit include collaboration and critical thinking.

